



Multidomain Line Sampling for Reliability and Sensitivity Analysis in Stochastic Linear Dynamics

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Abstract. This paper presents a framework for evaluating the first excursion probability and its sensitivity in linear dynamical structural systems subjected to Gaussian stochastic loading. The proposed approach involves multidomain Line Sampling, which is a technique that involves sampling along lines containing an infinite set of realizations of the stochastic loading. The numerical results demonstrate that this framework not only maintains accuracy but also exhibits superior computational efficiency compared to similar methodologies, making it a promising tool for reliability analysis.

Keywords: stochastic loading · linear dynamics · first-excursion probability · sensitivity · Line Sampling

1 Introduction

The first excursion probability quantifies the likelihood that one or more dynamic responses of a structural system, subjected to stochastic loading, will exceed a specified threshold within a given time period [16]. This probability serves as a critical measure of safety because it captures the chances that a system's response may cross a predefined limit, which could potentially indicate failure or loss of performance. The first excursion probability is influenced by two main factors: the characteristics of the stochastic loading and the properties of the structural system, such as the dimensions and material characteristics of its resisting elements. Understanding this probability is vital for engineers and designers because it helps ensure that structures can withstand unpredictable external forces while maintaining safe operational conditions.

Sensitivity analysis of the first excursion probability with respect to structural parameters provides additional value, as it reveals how changes in these parameters can affect the likelihood of threshold exceedance [12]. This information is invaluable for optimal design under uncertain conditions, allowing

for the identification of influential design variables and guiding decision-making to enhance system reliability. However, accurately calculating the first excursion probability and its sensitivity presents a substantial challenge, especially in practical scenarios where analytical solutions are typically unavailable, and simulation-based methods are required. These methods often involve high-dimensional problems and can be computationally intensive, making it necessary to explore efficient techniques for reliability analysis [14].

In response to these challenges, this paper introduces a comprehensive framework aimed at estimating the first excursion probability and its sensitivity, expressed as a gradient, for a specific class of structural dynamical systems—namely, deterministic linear structures subjected to Gaussian excitation. The proposed framework leverages a technique known as multidomain Line Sampling, which effectively reduces the number of required samples to achieve an accurate estimate of the probability [18]. This sampling method is designed to partition the input domain into multiple subdomains, which allows for more focused sampling and improved efficiency in capturing the critical regions that contribute to the first excursion event.

In addition to estimating the first excursion probability, the framework also evaluates its sensitivity as a byproduct of the reliability analysis. This requires performing a sensitivity analysis of the system's mode shapes and natural frequencies, as these dynamic properties significantly influence the system's response to stochastic excitation [11]. The paper includes a numerical example to demonstrate the practical application and effectiveness of the proposed framework. The results suggest that multidomain Line Sampling not only provides accurate estimates of the first excursion probability and its sensitivity but also achieves superior computational efficiency compared to other simulation-based methods [13, 17]. This makes the framework a promising tool for reliability analysis and optimization in the context of structural design under uncertainty.

2 Problem Characterization

2.1 Representation of Stochastic Loading

Assume that the dynamic loading applied to a structural system is modeled as a Gaussian process. This stochastic process has a duration of T and is represented using the Karhunen-Loève expansion. Consequently, the loading is denoted by $p(t, \mathbf{z})$, where p indicates the load magnitude, t represents time, and $\mathbf{z}$ is a vector of dimension n_z . This vector $\mathbf{z}$ is a realization of an n_z -dimensional standard normal distribution with a probability density function denoted as $f_{\mathbf{Z}}(\mathbf{z})$. Thus, the stochastic loading is represented as:

$$p(t, \mathbf{z}) = \mu_p(t) + \sum_{l=1}^{n_z} \psi_l(t) z_l \quad (1)$$

where $\mu_p(t)$ denotes the mean loading and $\psi_l(t)$, $l = 1, \dots, n_z$ are deterministic, time-dependent functions related with the covariance structure of the Gaussian process.

2.2 Stochastic Dynamic Response

Consider a structural system whose behavior is modeled as linear elastic and classically damped [4]. By employing a suitable numerical scheme (see, for example, [3]), the set of differential equations governing the structural behavior is given by:

$$\mathbf{M}(\mathbf{y})\ddot{\mathbf{x}}(t, \mathbf{y}, \mathbf{z}) + \mathbf{C}(\mathbf{y})\dot{\mathbf{x}}(t, \mathbf{y}, \mathbf{z}) + \mathbf{K}(\mathbf{y})\mathbf{x}(t, \mathbf{y}, \mathbf{z}) = \mathbf{g}(\mathbf{y})p(t, \mathbf{z}), \quad t \in [0, T] \quad (2)$$

where $\mathbf{x}$, $\dot{\mathbf{x}}$, and $\ddot{\mathbf{x}}$ are vectors representing the displacement, velocity, and acceleration, respectively, each of dimension $n_d \times 1$; $\mathbf{M}(\mathbf{y})$, $\mathbf{C}(\mathbf{y})$, and $\mathbf{K}(\mathbf{y})$ denote the mass, damping, and stiffness matrices, respectively, each of dimension $n_d \times n_d$, and each dependent on the parameter vector $\mathbf{y}$. The vector $\mathbf{g}(\mathbf{y})$, of dimension $n_d \times 1$, couples the stochastic loading $p(t, \mathbf{z})$ with the corresponding degrees of freedom of the structure. The vector $\mathbf{y}$ gathers parameters influencing structural behavior and may vary due to practical design decisions [8]. For instance, $\mathbf{y}$ could include quantities such as member thicknesses, material properties, and more.

Due to practical design considerations, it is assumed that there are n_r structural responses of interest. These responses are denoted as r_i , $i = 1, \dots, n_r$, and given the linearity of the structural system, they can be computed using a convolution integral between the corresponding unit impulse response functions h_i , $i = 1, \dots, n_r$, and the stochastic loading, expressed as:

$$r_i(t, \mathbf{y}, \mathbf{z}) = \int_0^t h_i(t - \tau, \mathbf{y}) p(\tau, \mathbf{z}) d\tau. \quad (3)$$

It should be noted that impulse response functions can be determined through modal analysis [4].

2.3 Reliability

In practical design scenarios, it is essential that the responses of a structural system remain below specific threshold levels, denoted as b_i , for all responses $i = 1, \dots, n_r$, over the duration T of the stochastic excitation. These threshold levels represent safety limits that, if exceeded, may indicate potential structural failure or compromised performance. The goal is to ensure that the structural responses stay within these limits to maintain safety, integrity, and serviceability during the entire period of loading.

However, these structural responses are influenced by both the parameter vector $\mathbf{y}$ – which includes design-related variables such as dimensions, material properties, and boundary conditions – and the realization $\mathbf{z}$ of the random variable vector associated with the Gaussian excitation. The vector $\mathbf{z}$ encapsulates the randomness or uncertainty present in the external loading, which could stem from unpredictable environmental factors like wind gusts, seismic activity, or other dynamic effects.

This dependence on $\mathbf{z}$ introduces uncertainty in the system's behavior, implying that even with carefully chosen design parameters $\mathbf{y}$, there remains a probability that the responses might exceed their prescribed threshold values during the duration T of the Gaussian loading. The critical measure used to quantify these chances is called the first excursion probability, denoted as p_F . The first excursion probability represents the likelihood that at least one response exceeds its threshold level within the given time frame.

Mathematically, this probability can be expressed using an integral over all possible realizations of $\mathbf{z}$, as follows:

$$p_F(\mathbf{y}) = \int_{\mathbf{z} \in \mathbb{R}^{n_z}} I_F(\mathbf{y}, \mathbf{z}) f_Z(\mathbf{z}) d\mathbf{z} \quad (4)$$

Here, $I_F(\mathbf{y}, \mathbf{z})$ is an indicator function that evaluates to one if any of the responses r_i exceed their respective threshold values b_i for a given combination of $\mathbf{y}$ and $\mathbf{z}$, and zero otherwise. This indicator function effectively captures whether a threshold violation occurs under a specific set of conditions.

For structural systems subjected to complex dynamic loads, the evaluation of the integral in Eq. (4) poses significant computational challenges. This difficulty arises primarily because the dimension n_z of the random variable vector $\mathbf{z}$ is typically large, often in the range of hundreds or even thousands, depending on the complexity of the stochastic model. Additionally, the indicator function $I_F(\cdot, \cdot)$ is not available in an analytical form but rather as a function known only at discrete points, making numerical evaluation necessary.

To overcome these challenges, advanced simulation techniques are employed, as traditional numerical integration methods are inefficient or even infeasible in high-dimensional settings. Methods such as Importance Sampling [2], Subset Simulation [1], or Line Sampling [10] have been proposed in the literature to efficiently approximate this probability.

2.4 Reliability Sensitivity

The failure probability integral, as expressed in Eq. (4), depends on the specific values that the parameter vector $\mathbf{y}$ assumes. This dependence aligns with physical intuition because altering the properties encapsulated within $\mathbf{y}$ – such as material characteristics, cross-sectional dimensions, or damping coefficients – directly impacts the behavior of the structural system. Consequently, any changes to $\mathbf{y}$ will likely influence the overall probability of failure. For instance, increasing the stiffness of certain structural elements could reduce deflections under loading, thereby lowering the likelihood of failure.

From a practical design perspective, understanding how variations in these parameters affect the failure probability is of great importance. Engineers and designers often aim to optimize structures to ensure reliability and safety under uncertain loading conditions. To achieve this, it is crucial to quantify the sensitivity of the failure probability with respect to each design parameter. This sensitivity analysis provides insights into which parameters have the most significant impact on structural reliability, enabling targeted improvements in design.

One common and effective approach for quantifying this sensitivity is to compute the gradient of the failure probability with respect to the parameters in $\mathbf{y}$. Mathematically, this involves taking the partial derivative of the failure probability $p_F(\mathbf{y})$ with respect to each component y_q of the vector $\mathbf{y}$, where $q = 1, \dots, n_y$ and n_y denotes the dimensionality of $\mathbf{y}$. The gradient of $p_F(\mathbf{y})$ is expressed as:

$$\frac{\partial p_F(\mathbf{y})}{\partial y_q} = \frac{\partial}{\partial y_q} \left(\int_{\mathbf{z} \in \mathbb{R}^{n_z}} I_F(\mathbf{y}, \mathbf{z}) f_{\mathbf{Z}}(\mathbf{z}) d\mathbf{z} \right), \quad q = 1, \dots, n_y. \quad (5)$$

In this expression, the gradient captures how small changes in each parameter y_q affect the failure probability. This information is crucial for identifying which parameters are most influential and for guiding design decisions to achieve greater structural resilience.

However, evaluating this gradient poses significant computational challenges. In many cases, calculating the gradient is just as difficult, if not more so, than estimating the failure probability itself. This is primarily due to the high dimensionality of the random variable vector $\mathbf{z}$, the complexity of the indicator function $I_F(\mathbf{y}, \mathbf{z})$, and the lack of closed-form solutions for the integral in Eq. (5). Advanced numerical techniques are often required to approximate the gradient efficiently, especially in high-dimensional settings.

3 Multidomain Line Sampling

3.1 Estimation of First Excursion Probability

The linearity of the responses of interest r_i with respect to the input Gaussian loading $p(t, \mathbf{z})$, as formulated in Eq. (3), leads to significant implications from a practical standpoint. This linearity simplifies the understanding of how the structural responses behave under stochastic excitation and allows for more efficient computation. As recognized and thoroughly discussed by [5], the realizations of $\mathbf{z}$ that lead to the exceedance of prescribed thresholds for the responses (referred to as the failure domain) lie within a region that is bounded by a series of hyperplanes. This concept is illustrated schematically in Fig. 1.

Several simulation methods have effectively utilized this geometric feature of the failure domain to calculate first excursion probabilities. Notable examples include Importance Sampling, Domain Decomposition Method, and Directional Importance Sampling, as explored in works such as [2, 9], and [13], respectively. These methods capitalize on the hyperplane-bounded nature of the failure domain to enhance the efficiency and accuracy of probability estimates.

One such technique, multidomain Line Sampling (mLS) [18], leverages this geometry by decomposing the failure probability p_F into a summation of multiple effective contributions, denoted as $p_{i,k}$. This decomposition is expressed as:

$$p_F(\mathbf{y}) = \sum_{i=1}^{n_r} \sum_{k=1}^{n_t} p_{i,k}(\mathbf{y}), \quad (6)$$

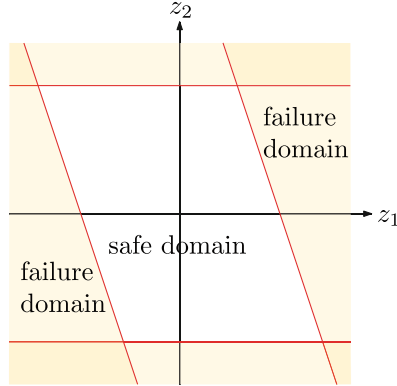


Fig. 1. Schematic representation of failure domain.

where each effective contribution $p_{i,k}$ is defined by the integral:

$$p_{i,k}(\mathbf{y}) = \int_{\mathbf{z} \in F_{i,k}} \frac{1}{\sum_{i_1=1}^{n_r} \sum_{k_1=1}^{n_t} I_{F_{i_1,k_1}}(\mathbf{y}, \mathbf{z})} f_Z(\mathbf{z}) d\mathbf{z}. \quad (7)$$

Here, $F_{i,k}$ represents the event in which the i -th response exceeds its prescribed threshold b_i at the k -th time instant. The term $I_{F_{i,k}}(\mathbf{y}, \mathbf{z})$ is an indicator function that takes the value of one if the realization $\mathbf{z}$ belongs to the event $F_{i,k}$, and zero otherwise. Notably, $F_{i,k}$ is bounded by two hyperplanes, as described in [5]. This geometric property of the failure domain forms the basis for efficient simulation techniques.

The effective contributions $p_{i,k}$ can be efficiently computed using the Line Sampling technique [10], which is illustrated schematically in Fig. 2. For simplicity, this figure considers the case where $n_r = 1$ and $n_t = n_z = 2$. In this illustration, $\boldsymbol{\alpha}_1$ denotes the important direction, which indicates the direction of the event $F_{1,1}$. The coordinates $z_1^{\parallel}$ and $z_1^{\perp}$ represent the directions parallel and orthogonal to the important direction $\boldsymbol{\alpha}_1$, respectively. The important direction is a vector of unit Euclidean norm pointing towards the failure domain $F_{1,1}$ [5].

In Line Sampling, a random sample is generated within the coordinate set $\mathbf{z}_1^{\perp}$ (depicted as a circle in Fig. 2). The values of the responses of interest are then computed along a line that passes through the selected sample point and is parallel to $\boldsymbol{\alpha}_1$ (illustrated with arrowheads in the figure). This approach significantly reduces computational effort, as only two dynamic analyses are required to calculate the responses along each line in view of linearity, making it numerically efficient [18].

In practice, when implementing multidomain Line Sampling, it is not necessary to explicitly calculate all effective contributions $p_{i,k}$ appearing in Eq. (6). Instead, the summation can be approximated using a simulation-based approach that introduces weights, denoted as $\omega_{i,k}$. This leads to the weighted summation:

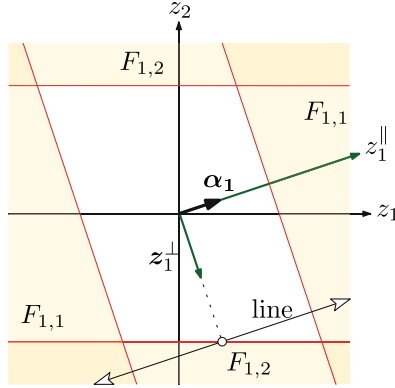


Fig. 2. Application of Line Sampling for estimating effective contribution associated with $p_{1,1}$.

$$p_F(\mathbf{y}) = \sum_{i=1}^{n_r} \sum_{k=1}^{n_t} \frac{1}{\omega_{i,k}} p_{i,k}(\mathbf{y}) \omega_{i,k}. \quad (8)$$

The weights $\omega_{i,k}$ are chosen to be proportional to the probability of occurrence of the event $F_{i,k}$, as suggested by [15] and [2]. The probability estimate is computed by first randomly sampling a pair (i, k) based on the weights and then estimating the effective contribution $p_{i,k}$ using Eq. (8) for a single randomly generated line. By repeating this procedure N times and averaging the results obtained from the N lines, both the failure probability and its coefficient of variation can be estimated, as described in [18].

This weighted approach enhances computational efficiency by focusing resources on more critical regions of the failure domain, thereby providing accurate estimates of failure probability with reduced numerical effort.

3.2 Estimation of Gradient of the First Excursion Probability

The gradient of the failure probability can be determined by differentiating Eq. (6) with respect to the parameter y_q , for $q = 1, \dots, n_y$. This differentiation yields:

$$\frac{\partial p_F(\mathbf{y})}{\partial y_q} = \sum_{i=1}^{n_r} \sum_{k=1}^{n_t} \frac{1}{\omega_{i,k}} \frac{\partial p_{i,k}(\mathbf{y})}{\partial y_q} \omega_{i,k}. \quad (9)$$

The central challenge here lies in calculating the partial derivative of each effective contribution $p_{i,k}$, as defined in Eq. (7), with respect to the design parameter y_q . This differentiation requires careful consideration because each $p_{i,k}$ is defined as an integral over a region in the random variable space. To address this challenge, one can apply Leibniz's rule for differentiating an integral with

respect to a parameter that appears in both the integrand and the integration limits, as detailed in [6].

When Leibniz's rule is applied to differentiate Eq. (7), the task of calculating the derivative with respect to y_q is reduced to evaluating the sensitivity of the responses r_i with respect to y_q . This reduction occurs because the responses are expressed as convolution integrals with the stochastic loading, as shown in Eq. (3). Therefore, determining the gradient of the failure probability ultimately depends on calculating the sensitivity of the impulse response functions h_i , for $i = 1, \dots, n_r$, with respect to the parameter y_q .

Impulse response functions are derived from the system's spectral properties, which include the mode shapes and natural frequencies of the structural system. These spectral properties are directly influenced by the design parameters in $\mathbf{y}$, such as stiffness, mass distribution, and damping characteristics. Thus, the problem of determining the gradient of the failure probability is effectively transformed into the problem of finding the sensitivity of the system's spectral properties with respect to the design parameters. This is a well-researched problem in structural dynamics, with several methodologies developed over the years to address it; see, for example, the work by [7].

In this contribution, the sensitivity of the spectral properties is computed using the approach proposed in [11]. This method offers significant numerical advantages, as it avoids the need for repeated spectral analysis for each parameter variation. Instead, it efficiently calculates sensitivities by solving systems of linear equations associated with the structural matrices and their derivatives with respect to y_q . These structural matrices include the mass, damping, and stiffness matrices, which characterize the dynamic behavior of the system.

To summarize, the key steps in calculating the gradient of the failure probability involve: (1) applying Leibniz's rule to differentiate the integral in Eq. (7) with respect to the design parameters, (2) computing the sensitivity of the impulse response functions using modal analysis, and (3) determining the derivative of the spectral properties, such as natural frequencies and mode shapes. The proposed approach, based on the method by [11], significantly reduces the computational burden by leveraging efficient numerical techniques instead of repetitive spectral computations. This efficiency is crucial for practical applications, where large-scale structural systems and multiple design parameters are often involved.

4 Example

The application of multidomain Line Sampling for estimating the first excursion probability and its gradient is demonstrated using an example involving a linear elastic single-degree-of-freedom oscillator, as depicted in Fig. 3.

The system under consideration has mass $m = 1$ [kg], stiffness $k = 4\pi^2$ [N/m] and damping coefficient $d = 2\%$. The system is subjected to a stochastic ground acceleration, modeled as a discrete white noise process. The spectral intensity is $S = 1$ [m²/s³], duration $T = 15$ [s] and time step $\Delta t = 0.01$ [s]. It is

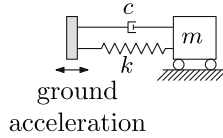


Fig. 3. Single-degree-of-freedom system.

considered that the system fails when its absolute displacement exceeds 2.22 [m]. The objective of this example is to estimate the failure probability and its sensitivity with respect to the stiffness k .

The estimates of the first excursion probability and its sensitivity obtained applying multidomain Line Sampling (mLS) are shown in Figs. 4 and 5. These Figures present in addition the results obtained with Directional Importance Sampling (DIS), which is another simulation approach that allows estimating the sought first excursion probability and its sensitivity [13,17]. The obtained results show the evolution of the estimators as a function of the number of dynamic analyses N . The results produced with mLS exhibit less variability than those produce with DIS. This highlights the potential of mLS as an effective simulation tool for calculating first excursion probability and its sensitivity in stochastic linear dynamics. Furthermore, from Fig. 5, it is noted that the sensitivity of the

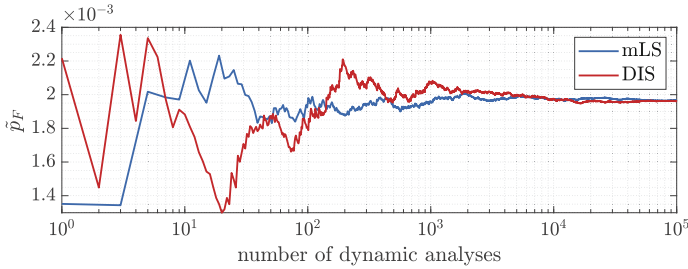


Fig. 4. Estimator for the first excursion probability.

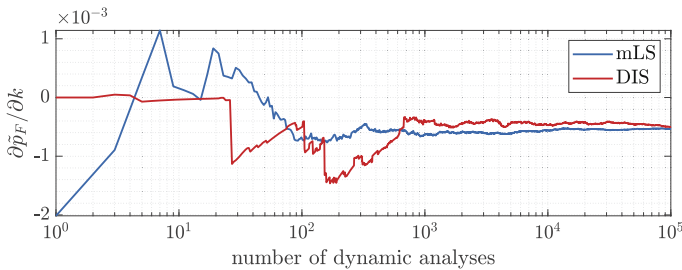


Fig. 5. Estimator for the sensitivity of the first excursion probability with respect to stiffness.

failure probability is negative. This is reasonable from a physical viewpoint, as a larger stiffness allows decreasing displacement.

5 Conclusions

The results presented in this paper indicate that the sensitivity of the first excursion probability can be effectively calculated using multidomain Line Sampling (mLS). The proposed mLS approach demonstrates promising capabilities in efficiently estimating both the failure probability and its gradient with respect to critical design parameters. This capability is particularly valuable for structural reliability analysis, where understanding how changes in structural properties influence the likelihood of failure is crucial for optimizing design and enhancing safety under uncertain loading conditions.

Moreover, the numerical results suggest that mLS may offer significant advantages over traditional methods such as Directional Importance Sampling (DIS). DIS is a well-established technique used for estimating failure probabilities, especially in high-dimensional settings.

While the findings reported here are encouraging, further research is needed to validate these conclusions across a broader range of scenarios. Specifically, future studies will focus on examining the effectiveness of mLS in cases involving different types of Gaussian excitations beyond discrete white noise processes. Examples of more general Gaussian excitations include stationary processes with specific spectral characteristics, colored noise with correlated components, and non-stationary excitations that evolve over time. These more complex excitation models better reflect real-world dynamic loading scenarios, such as seismic events, wind-induced vibrations, or ocean waves.

In addition, extending the application of mLS to more complex structural systems is a key area for future investigation. Real-world structures often feature multiple degrees of freedom, nonlinearity, or coupled responses, which present additional challenges for reliability analysis. Validating mLS in such contexts would demonstrate its robustness and applicability to a wider range of engineering problems, including those involving multi-modal responses or interacting components.

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