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Efficient slope reliability analysis under soil spatial variability

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Abstract

Geotechnical engineering confronts different sources of uncertainty, which can be suitably described by random fields. Therefore, in recent years, random field theory has been commonly used to describe spatial variability of natural soils. The main methods used to propagate these uncertainties are direct Monte Carlo simulations, approximation methods, surrogate model methods, and sampling methods [1]. The direct Monte Carlo simulation is often used to face engineering problems with a high probability of failure ($10^{-1} - 10^{-3}$) due to its assured convergence. However, Monte Carlo exhibits significant constraints when applied to problems entailing a low probability of failure ($< 10^{-3}$) because of the excessive number of simulation required. To overcome these limitations, advanced simulation methods have been developed because of their improved accuracy, efficiency, and robustness with respect to dimensionality. However, there is no unique and efficient method to deal with this issue up to the present time. In this paper, the Latinized partially stratified sampling (LPSS) is employed to estimate the failure probability of slopes considering spatial variability using the maximum entropy distribution with fractional moments (MEDFM) [2,3]. In a first step, Karhunen-Loève (K-L) expansion is used to characterize soil properties through random fields. K-L expansion is performed for the considered random field characteristics (mean value, variance, autocorrelation function and scales of fluctuations). In a second step, the advantages of LPSS and the basics of MEDFM are presented. Then, the specific implementation procedures of the present approach based on LPSS and MEDFM are described. Finally, the proposed approach is used to analyze a case study of an undrained slope with random fields of strength parameters [4].

In this application, the undrained clay slope is used to illustrate a specific implementation of the approach [5]. The geometry of the scenario considered is given in Fig. 1. The total slope height in this example is 10 m, and its length is 30 m. The slope is modeled via a 2d finite element implemented in SIGMA/W and SLOPE/W, where the mesh types are 4-node quadrilateral and 3-node triangular meshes. Because of the geometry of the slope, most of the finite elements are square, while some degenerate into triangles. Fig. 1 show the meshed finite element, consisting of 910 elements and 981 nodes. For illustrative purposes, a conventional elastic and perfect plastic model based on the Mohr-Coulomb failure criterion is used to represent the stress-strain behavior of the soil. The boundary conditions are fully fixed on the vertical ends of the bottom and left sides of the model.

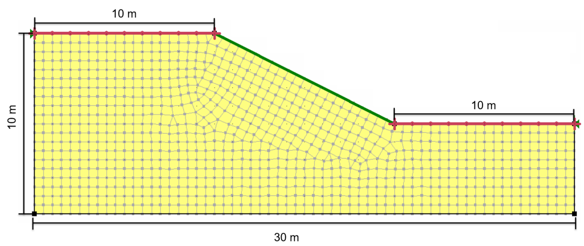

Figure 1: Finite element model for slope in example 1

In this analysis, the cohesion is considered as uncertain, where the shear strength is modeled with a random field with log-normal distribution, while the angle of internal friction is assumed to be certain and equal to 0. Additionally, the Young's modulus, Poisson's ratio, and the soil unit weight are considered in this example as deterministic quantities. This is an assumption motivated by their limited influence compared to the cohesion. Table 1 summarizes the statistical properties of the soil parameters for this considered slope.

Three main steps are performed to obtain a random field realization of the undrained shear strength with spatial variability. First, an independent standard normal sample matrix of dimension 80×900 is generated using LPSS. This is because 900 sets of random samples are sufficient for convergence for slope engineering with that failure probability. The 80-dimensional random variables

Table 1: Material parameters in the example

Parameter	Mean value	COV
Unit weight (kN/m^3)	20	- (deterministic)
Youngs modulus (MPa)	100	- (deterministic)
Poissons ratio	0.3	- (deterministic)
Cohesion (kPa)	23	0.3
Friction angle (°)	-	- (deterministic)

of LPSS are generated by creating 40 groups of two random variables each. Then, the parameter matrix of the undrained shear strength in the physical space is obtained by K-L expansion. Finally, the sampled values of cohesion are assigned to each element separately. After deterministic analysis, the LPSS-MEDFM method was used to calculate the probability of failure of this slope engineering problem. The resulting PDF of f_s and CDF of f_s are shown in Figs. 2 and 3, respectively. The probability of failure is the vertical ordinate of the CDF corresponding to when f_s is 1. The PDF and CDF calculated by the proposed approach in this paper are also consistent with the Monte Carlo method.

In addition, the results of the approach were compared with some literature results. Among them, there are results obtained by LHS and MCS with MEDFM. For MCS-MEDFM and LHS-MEDFM, it requires 2000 and 1600 calculations, respectively. For LPSS-MEDFM, the results were calculated for 400, 900, 1600, 2500 and 3600 samples, respectively. It was found that only 900 deterministic calculations were needed to obtain reliable failure probability results. These results indicate that the proposed approach can produce sufficiently accurate failure probability for smaller samples. To investigate the applicability of the method for small failure probabilities, we considered a scenario of undrained clay slope for a coefficient of variation of 0.15. It was found that 3600 deterministic calculations were needed to obtain the failure probability result of 1.5e-4. This result shows that the method is very effective for small failure probabilities.

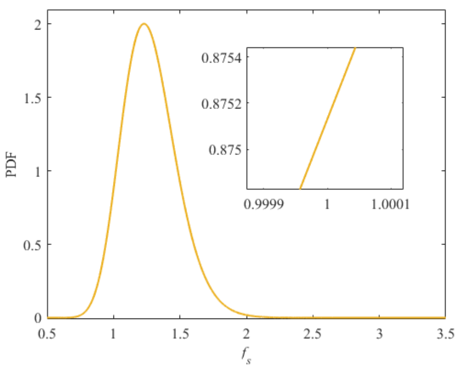

Figure 2: The result of PDF in the example

It is found that the proposed approach has high efficiency for geotechnical engineering problems with spatial variability. This is because the LPSS method has good efficiency and applicability for high-dimensional sampling problems compared to other methods. The proposed approach is suitable for geotechnical engineering problems with a low probability of failure because the MEDFM method can obtain PDFs of safety factors from data with statistical information. This is because moments of fractional order include more information than moments of integer order. Combining fractional order moments with MED makes the approach more efficient.

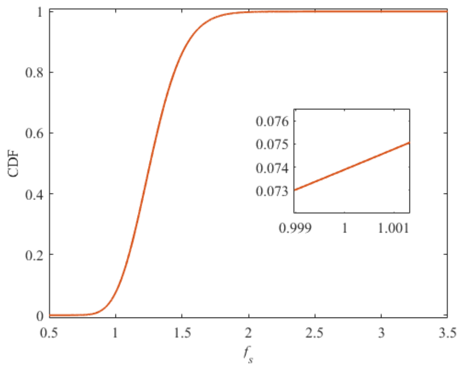

Figure 3: The CDF result in the example

Key words: Slope; Random field; Reliability analysis; Maximum entropy distribution; Latinized partial stratified sampling

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