

## First Excursion Probability Estimation of a Bilinear Conservative Oscillator Subject to Gaussian Loading

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### Abstract

The study of oscillators under stochastic loading is essential in fields such as mechanical, ocean, wind, and earthquake engineering. The uncertainty arising from the intrinsic nature of the loading can be quantified using the first excursion probability, which measures the likelihood that an oscillator's response will exceed a specified threshold during stochastic excitation. However, estimating this probability is challenging due to the need to manage a large number of random variables to represent the load, as well as potential nonlinearities in the restoring force and the non-stationary nature of both the loading and the oscillator's response.

This work presents an efficient method for estimating first excursion probabilities for nonlinear oscillators subjected to Gaussian loading, particularly for systems with a bilinear, conservative restoring force. The technique focuses on exceedance probabilities within the nonlinear response range by dividing the calculation into two components: first, estimating the probability of failure in the elastic range, and then assessing the probability of failure in the inelastic range.

To estimate the probability within the elastic range, an advanced simulation-based variance reduction method is employed. This method effectively addresses scenarios where the oscillator's response remains linear. This technique also generates samples of the oscillator's maximum response in the inelastic range, which are subsequently used to construct an extreme value distribution for the inelastic maximum response. By synthesizing the response data, this distribution facilitates a more accurate and efficient estimation of first excursion probabilities across both response ranges. In this context, the variance reduction sampling method is optimally utilized to explore both elastic and inelastic ranges, while the extreme value distribution leverages this information to enhance the estimation of the first excursion probability. A numerical example is provided to illustrate the application of the proposed approach.

Key words: bilinear conservative oscillator, Gaussian loading, extreme value distribution, advanced simulation method