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First Excursion Probability Estimation of a Bilinear Conservative Oscillator Subject to Stochastic Gaussian Loading

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Abstract

Characterization of the behavior of oscillators subject to stochastic loading is of paramount importance in several practical applications of mechanical, ocean, wind, and earthquake engineering, among others. This characterization can be carried out in terms of probabilistic descriptors such as second-order statistics or probability distributions. One probabilistic descriptor of much relevance is the first excursion probability, that measures the chances that the oscillator's response exceeds a prescribed threshold within the duration of the stochastic loading. The calculation of such excursion probability is usually far from trivial, as it is necessary to:

- Cope with a large number of random variables to represent the stochastic loading.
- Consider possible nonlinearities with respect to the restoring force of the oscillator.
- Address non-stationarity of both the loading and the oscillator's response.

Different approaches have been developed for estimating first excursion probabilities of nonlinear oscillators, such as path integration [1,4], probability density evolution methods [5], extreme value distribution [2,7] and simulation [3], to name a few. Despite all the ongoing research in this area, there is still a need for approaches which exhibit a high numerical efficiency, and which offer sufficient precision.

Considering the context described previously, this contribution proposes an approach to estimate first excursion probabilities of nonlinear oscillators subject to stochastic loading. The focus is on a specific class of problems, namely an oscillator which possesses a bilinear, conservative restoring force which is subject to stochastic forces of the Gaussian type. The proposed approach is geared towards the calculation of probabilities of exceedance within the nonlinear range of the response. For this purpose, the task of calculating the failure probability within the nonlinear range is split into two tasks: estimating the probability of failure associated with the elastic response and then, estimating the first excursion probability in the inelastic range [3,7]. In fact, the probability of failure is expressed as:

$$P[F] = P[F_e]P[F|F_e] \tag{1}$$

where $P[\cdot]$ denotes probability of the event between brackets, F_e denotes the excursion event associated with the elastic limit and F denotes the excursion event associated with the inelastic range. The first excursion probability associated with the elastic range $P[F_e]$ is calculated with Directional Importance Sampling [6], which is a simulation method specially tailored for cases where the restoring force behaves within the linear range. In addition, and as a by-product of Directional Importance Sampling, samples of the maximum value of the response of the oscillator are generated within the inelastic range. These samples are then post-processed in the form of statistical moments to construct the extreme value distribution of the maximum response within the inelastic range [2,7]. In such a way, it is possible to make optimal use of Directional Importance Sampling for exploring the elastic and inelastic ranges of the response, while extreme value distribution can synthesize the information collected within the inelastic range to render the estimation of first excursion probabilities tractable. A numerical example illustrates the application of the proposed approach.

Key words: bilinear conservative oscillator; Gaussian loading; extreme value distribution; Directional Importance Sampling.

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