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# Reliability analysis of Sequential Series Systems by Bayesian active learning with multiple Gaussian process models

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Many engineering systems exhibit a sequential series structure, in which subsystem outputs are propagated downstream and failure occurs if any limit state is violated. Such systems pose significant challenges to conventional surrogate modeling, as uncertainty must be tracked not only within each subsystem but also through the entire dependency chain. This work presents a hierarchical active learning probabilistic integration framework for efficient reliability analysis of sequential series systems. Each subsystem is represented by a Gaussian Process surrogate, enabling flexible modeling of subsystem behavior across stages. We introduce a new uncertainty-based acquisition function that combines local classification uncertainty with the upstream-propagated input uncertainty, ensuring that only samples with significant influence on the failure-probability estimate are selected. A finite element case study demonstrates that the proposed method significantly reduces the number of model evaluations while maintaining high accuracy and robustness compared with state-of-the-art all-in-one approaches. The framework provides an efficient and robust solution for reliability assessment of complex multilevel systems.

***Keywords***: System reliability analysis, Sequential series system, Bayesian inference, Active learning, Gaussian process.

## 1 Introduction

Reliability analysis aims to quantify the likelihood of undesired system performance by explicitly accounting for all relevant sources of uncertainty. Existing reliability analysis methods can be roughly classified into three major groups, asymptotic approximation (Rackwitz and Fiessler 1978, Breitung 1984), stochastic simulation (Melchers 1989, Fishman 2013) and surrogate-assisted methods (Kaymaz 2005, Blatman and Sudret 2010). Surrogate models with active learning strategies further enhance both the accuracy and efficiency of reliability analysis (Echard et al. 2011). Dang et al. (2022) interprets probability estimation within the Bayesian Failure Probability Inference (BFPI) framework, leading to Bayesian active learning methods such as Active Learning Probabilistic Integration (ALPI) (Dang et al. 2021), weakly Bayesian active learning quadrature (Dang et al. 2025), etc.

Practical engineering systems can be configured in various forms, and their failure behaviors are often highly complex due to interactions among multiple subsystems. Classical configurations include series systems (Chen 2023), parallel systems (Chen 2025), and mixed series-parallel systems (Zhang and Du 2024). A particular class referred to as sequential systems, in which subsystems are connected in a cascade manner through linking variables. Limited research has been devoted to the reliability analysis of sequential systems. Constant et al. (2025) employs an imputation criterion to avoid unnecessary evaluations of the costly upstream model. Ye et al. (2025) provides an efficient framework for the reliability analysis of nested engineering systems. Xu et al. (2020) introduced a multiscale uncertainty analysis framework that efficiently propagates uncertainties across scales. In this work, we consider a *first-order sequential series system*. Here, *sequential series* refers to a multi-failure-mode configuration in which subsystems are arranged sequentially and system failure occurs if any limit state is violated. The term *first-order* indicates that adjacent subsystems are coupled through a single scalar linking variable. Within this setting, we propose a Hierarchical Active Learning Probabilistic Integration (HALPI) framework.

The main contributions of this study can be summarized as follows. First, based on the BFPI, a hierarchical method is proposed to estimate the stage-wise failure probabilities of first-order sequential series systems. Second, a new acquisition function is developed to guide the selection of which point and which subsystem to evaluate. Third, a stopping criterion is developed based on the quantified prediction uncertainty of each Gaussian Process (GP) model to ensure a reliable convergence of the algorithm.

The remainder of this article is organized as follows. Section 2 formulates the reliability problem for first-order sequential series systems. Section 3 extends the BFPI framework to first-order sequential systems and analyzes the prediction uncertainty measures. Section 4 presents the proposed HALPI method, including the new learning function, the new stopping criterion and the active–passive update mechanism. Section 5 provides an engineering example to demonstrate the accuracy, robustness, and computational efficiency of the proposed method in comparison with existing approaches. Section 6 concludes the study and discusses future research directions.

## 2 Reliability analysis of first-order sequential series systems

Consider a vector of input random variables $\mathbf{X}=[X_1,X_2,\cdots,X_n]^\top \in D_{\mathbf{X}} \subseteq \mathbb{R}^n$ of all performance functions and all subsystems. And let $\mathbf{x}=[x_1,x_2,\cdots,x_n]^\top$ denote a generic realization of this random vector in its domain $D_{\mathbf{X}}$. In this work, a vector indexed by a set denotes the corresponding subvector. With this notation, a first-order sequential system can be expressed as the hierarchical mapping

$$y_1 = M_1(\mathbf{x}_{\boldsymbol{\alpha}_1}),\ y_i = M_i(\mathbf{x}_{\boldsymbol{\alpha}_i}, y_{i-1}),\ i\in[2,\cdots,N], \tag{1}$$

where $M_i$ is a computational model that maps inputs to $y_i$, the response of subsystem $i$; $\boldsymbol{\alpha}_i \subseteq \{1,\ldots,n\}$ includes the indices relevant to inputs of subsystem $i$. In a first-order sequential series system, each subsystem possesses a distinct failure mode, characterized by its associated performance function, $g_i = S_i(\mathbf{x}_{\boldsymbol{\beta}_i}, y_i) = S_i'(\mathbf{z}_i)$, where $\boldsymbol{\beta}_i \subseteq \{1,\ldots,n\}$ denotes the set of indices involved only in the $i$-th performance function. $\mathbf{z}_i$ is $\mathbf{x}_{\boldsymbol{\gamma}_i}$ if $i=1$, is $(\mathbf{x}_{\boldsymbol{\gamma}_i}, y_{i-1})$ otherwise, where $\boldsymbol{\gamma}_i = \boldsymbol{\alpha}_\mathbf{i} \cup \boldsymbol{\beta}_i$. The failure probability associated with the first $i$ stages is

$$P_{F_i} = \int_{D_\mathbf{X}} I_{1:i}(\mathbf{x}_{\boldsymbol{\gamma}_{1:i}}) f_{1:i}(\mathbf{x}_{\boldsymbol{\gamma}_{1:i}}) d\mathbf{x}_{\boldsymbol{\gamma}_{1:i}};\quad I_{1:i}(\mathbf{x}_{\boldsymbol{\gamma}_{1:i}}) = \begin{cases} 1, & \text{if } \exists k\in\{1,\ldots,i\} \text{ s.t. } g_k \le 0, \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

where $I_{1:i}(\mathbf{x}_{\boldsymbol{\gamma}_{1:i}})$ is the series indicator function; $\boldsymbol{\gamma}_{1:i} = \cup_{j=1}^{i} \boldsymbol{\gamma}_j$; $f_{1:i}(\cdot)$ denotes the joint PDF of $\mathbf{X}_{\boldsymbol{\gamma}_{1:i}}$.

## 3 The proposed hierarchical BFPI framework

### 3.1 *Prior and Posterior distributions of $i$-th GP model*

Due to its high computational cost, the $i$-th performance function is learned by a GP with following prior:

$$G_{i,0}(\mathbf{z}_i) \sim \mathrm{GP}\big(\mu_{i,0}(\mathbf{z}_i), c_{i,0}(\mathbf{z}_i, \mathbf{z}_i')\big). \tag{3}$$

In this work, a constant mean function together with a squared exponential kernel is adopted. The hyperparameters are collected in the vector $\boldsymbol{\theta}_i$. Assume that we obtain a training dataset $\mathbf{D}_i = \{\mathcal{Z}_i, \mathbf{g}_i\}$, where $\mathcal{Z}_i = \left[\mathbf{z}_i^{(1)};\cdots;\mathbf{z}_i^{(d_i)}\right]$ and $\mathbf{z}_i^{(k)} \in \mathbb{R}^{1\times n_i}$; $\mathbf{g}_i = \left[g_i^{(1)};\cdots;g_i^{(d_i)}\right]$ and $g_i^{(k)} \in \mathbb{R}$. Given $\mathbf{D}_i$, the hyperparameters $\boldsymbol{\theta}_i$ are estimated via maximum likelihood estimation (Frazier 2018). Conditional on $\mathbf{D}_i$, the posterior distribution of $g_i$ follows a GP:

$$G_i(\mathbf{z}_i) \sim \mathrm{GP}\big(\mu_i(\mathbf{z}_i), c_i(\mathbf{z}_i, \mathbf{z}_i')\big). \tag{4}$$

For any input $\mathbf{z}_i$, the posterior mean and covariance functions are given by:

$$\begin{aligned} \mu_i(\mathbf{z}_i) &= \mu_{i,0}(\mathbf{z}_i) + \mathbf{c}_{i,0}(\mathbf{z}_i, \mathcal{Z}_i)^\top \mathbf{C}_{i,0}^{-1}(\mathbf{g}_i - \boldsymbol{\mu}_{i,0}(\mathcal{Z}_i)), \\ c_i(\mathbf{z}_i, \mathbf{z}_i') &= c_{i,0}(\mathbf{z}_i, \mathbf{z}_i') - \mathbf{c}_{i,0}(\mathbf{z}_i, \mathcal{Z}_i)^\top \mathbf{C}_{i,0}^{-1} \mathbf{c}_{i,0}(\mathbf{z}_i', \mathcal{Z}_i), \end{aligned} \tag{5}$$

After all GPs are trained, the predictive performance values are obtained recursively as

$$\hat{g}_i = \mathbb{E}[G_i(\hat{\mathbf{z}}_i)],\qquad \hat{\mathbf{z}}_i = \hat{T}_i(\mathbf{x}_{\boldsymbol{\gamma}_{1:i}}) = \begin{cases} \mathbf{x}_{\boldsymbol{\gamma}_i}, & \text{if } i=1, \\ (\mathbf{x}_{\boldsymbol{\gamma}_i}, \hat{y}_{i-1}), & \text{otherwise,} \end{cases} \tag{6}$$

with the estimated subsystem response $\hat{y}_i = S_i^{-1}(\mathbf{x}_{\boldsymbol{\beta}_i}, \hat{g}_i)$, where $S_i^{-1}(\mathbf{x}_{\boldsymbol{\beta}_i}, \cdot)$ denotes the inverse mapping of $S_i(\mathbf{x}_{\boldsymbol{\beta}_i}, \cdot)$, which is assumed to be bijective for the sake of simplicity.

The posterior distribution of the failure probability of stage $i$ under the HBFPI framework is

$$\hat{P}_{F_i} = \int_{D_\mathbf{X}} \hat{I}_{1:i}(\mathbf{x}_{\boldsymbol{\gamma}_{1:i}}) f_{1:i}(\mathbf{x}_{\boldsymbol{\gamma}_{1:i}}) \mathrm{d}\mathbf{x}_{\boldsymbol{\gamma}_{1:i}} \tag{7}$$

where $\hat{I}_{1:i}$ follows a generalized Bernoulli process. Its posterior mean is:

$$\mathbb{E}\Big[\hat{I}_{1:i}(\mathbf{x}_{\gamma_{1:i}})\Big] \approx 1-\prod_{k=1}^{i}\Big(1-\mathbb{P}\Big[\hat{I}_k(\hat{\mathbf{z}}_k)=1\Big]\Big) = 1-\prod_{k=1}^{i}\left(1-\Phi\left(\frac{-\mathbb{E}[G_k(\hat{T}_k(\mathbf{x}_{\gamma_{1:k}})]}{\sqrt{\mathbb{V}[G_k(\hat{T}_k(\mathbf{x}_{\gamma_{1:k}}))]}}\right)\right) = h_i(\mathbf{x}_{\gamma_{1:k}}), \tag{8}$$

where $\hat{I}_k$ is the posterior distribution of the indicator function of the $k$-th performance function. Such a result can be obtained by assuming independence of the components of the underlying series system. Results shown in Section 5 indicate that this approximation has a limited impact on the accuracy and efficiency of the proposed method.

Mean of $\hat{P}_{F_i}$ serves as the estimate of the failure probability and is given by:

$$\mathbb{E}[\hat{P}_{F_i}] = \mathbb{E}\left[\int_{D_{\mathbf{X}}} \hat{I}_{1:i}(\mathbf{x}_{\gamma_{1:i}}) f_{1:i}(\mathbf{x}_{\gamma_{1:i}})\mathrm{d}\mathbf{x}_{\gamma_{1:i}}\right] \approx \mathbb{E}_{\mathbf{X}}\Big[h_i(\mathbf{X}_{\gamma_{1:k}})\Big]. \tag{9}$$

The corresponding upper bound of coefficient of variation (C.o.V) is:

$$\overline{\mathrm{CoV}}[\hat{P}_{F_i}] = \frac{\sqrt{\overline{\mathbb{V}}[\hat{P}_{F_i}]}}{\mathbb{E}[\hat{P}_{F_i}]} = \frac{\mathbb{E}_{\mathbf{X}}\left[\sqrt{\mathbb{V}\Big[\hat{I}_{1:i}(\mathbf{X}_{\gamma_{1:i}})\Big]}\right]}{\mathbb{E}[\hat{P}_{F_i}]}. \tag{10}$$

### 3.2 *Measure of propagated prediction uncertainty*

For $i>1$, we introduce the augmented input:

$$\mathbf{z}_i^G = T_i^G(\mathbf{x}_{\gamma_{1:i}}) = \Big[\mathbf{x}_{\gamma_i}, Y_{i-1}^G(\hat{\mathbf{z}}_{i-1})\Big], \quad Y_{i-1}^G(\hat{\mathbf{z}}_{i-1}) = S_{i-1}^{-1}\Big(\mathbf{x}_{\boldsymbol{\beta}_{i-1}}, G_{i-1}(\hat{\mathbf{z}}_{i-1})\Big), \tag{11}$$

where $Y_{i-1}^G(\hat{\mathbf{z}}_{i-1})$ is a random variable. Just for simplicity we denote $\mathcal{Y} = Y_{i-1}^G(\hat{\mathbf{z}}_{i-1})$. Conditioned on $\mathcal{Y}$, $\hat{I}_i(\mathbf{z}_i^G \mid \mathcal{Y})$ the indicator function of the performance function $i$ is a generalized Bernoulli process with posterior mean

$$P_i(\mathbf{z}_i^G \mid \mathcal{Y}) = \Phi\left(\frac{-\mathbb{E}[G_i(\mathbf{z}_i^G)]}{\sqrt{\mathbb{V}[G_i(\mathbf{z}_i^G)]}}\Big|\mathcal{Y}\right). \tag{12}$$

The variance of $\hat{I}_i(\mathbf{z}_i^G)$ is given by:

$$\mathbb{V}[\hat{I}_i(\mathbf{z}_i^G)] = \mathbb{E}_{\mathcal{Y}}[P_i(\mathbf{z}_i^G \mid \mathcal{Y}) - P_i^2(\mathbf{z}_i^G \mid \mathcal{Y})] + \mathbb{V}_{\mathcal{Y}}[P_i(\mathbf{z}_i^G \mid \mathcal{Y})]. \tag{13}$$

The second term defined in the following as $\mathrm{VP}_i(\mathbf{z}_i^G) = \mathbb{V}_{\mathcal{Y}}\Big[P_i(\mathbf{z}_i^G \mid \mathcal{Y})\Big]$, reflects the prediction uncertainty propagated from the $(i-1)$-th GP to the $i$-th GP.

## 4 The proposed HALPI method

### 4.1 *Active-passive learning mechanism*

The acquisition function for the $i$-th GP is defined as

$$L_i(\mathbf{x}_{\gamma_{1:i}}) = \frac{\mathrm{SVP}_i(\mathbf{x}_{\gamma_{1:i}})}{\max_{\mathbf{x}'_{\gamma_{1:i}}\in\mathbb{R}} \mathrm{SVP}_i(\mathbf{x}'_{\gamma_{1:i}})} + \frac{\sqrt{\mathbb{V}\Big[\hat{I}_{1:i}(\mathbf{x}_{\gamma_{1:i}})\Big]}}{\max_{\mathbf{x}'_{\gamma_{1:i}}\in\mathbb{R}} \sqrt{\mathbb{V}\Big[\hat{I}_{1:i}(\mathbf{x}'_{\gamma_{1:i}})\Big]}}; \quad \mathrm{SVP}_i(\mathbf{x}_{\gamma_{1:i}}) = \begin{cases} 0, & \text{If } i=1, \\ \sum_{k=2}^{i} \mathrm{VP}_k(T_k^G(\mathbf{x}_{\gamma_{1:k}})), & \text{Otherwise,} \end{cases} \tag{14}$$

The most informative sample for $i$-th GP is $\mathbf{x}^{*,i} = \arg\max_{\mathbf{x}_{\gamma_{1:i}}\in D_{\mathbf{X}}} L_i(\mathbf{x}_{\gamma_{1:i}})$. The inputs of the first $k$ GP models are extracted from $\mathbf{x}^{*,i}$ and are denoted as $\mathbf{x}_{\gamma_{1:k}}^{*,i} \subset \mathbf{x}^{*,i}$, $(k\in[1,i-1])$. Then, the $k$-th linking variable can be obtained by

$$y_k^{\Delta} = M_k^S(\mathbf{x}_{\gamma_{1:k}}^{*,i}) = \begin{cases} S_k^{-1}\Big(\mathbf{x}_{\boldsymbol{\beta}_k}^{*,i}, \mathbb{E}[G_k(\mathbf{z}_k^{\Delta})]\Big) & \text{, if } \mathrm{VP}_k(\mathbf{z}_k^{G,\Delta}) \le \mathrm{VP}_{\mathrm{tol}} \\ M_1(\mathbf{x}_{\boldsymbol{\alpha}_1}^{*,i}) & \text{, if } \mathrm{VP}_1(\mathbf{z}_1^{G,\Delta}) > \mathrm{VP}_{\mathrm{tol}} \text{ and } k=1 \\ M_k(\mathbf{x}_{\boldsymbol{\alpha}_k}^{*,i}, y_{k-1}^{\Delta}) & \text{, if } \mathrm{VP}_k(\mathbf{z}_k^{G,\Delta}) > \mathrm{VP}_{\mathrm{tol}} \text{ and } k>1, \end{cases} \tag{15}$$

where

$$\mathbf{z}_k^{\Delta} = T_k^{\Delta}(\mathbf{x}_{\gamma_{1:k}}^{*,i}) = \begin{cases} \mathbf{x}_{\gamma_1}^{*,i}, & \text{if } k = 1, \\ \left[\mathbf{x}_{\gamma_k}^{*,i}, y_{k-1}^{\Delta}\right], & \text{otherwise,} \end{cases} \tag{16}$$

is the input of the $k$-th GP and $\mathbf{z}_k^{G,\Delta}$ is the corresponding augmented input. For the example presented in this paper, $\mathrm{VP}_{\mathrm{tol}} = 0.05^2$ is used. If the original model is called, the $k$-th GP model is updated using the new pair $\{\mathbf{z}_k^{\Delta}, g_k^{\Delta}\}$, where $g_k^{\Delta} = S_k(\mathbf{x}_{\boldsymbol{\beta}_k}^{*,i}, y_k^{\Delta})$. These pairs arise as byproducts when obtaining the training pair $\{\mathbf{z}_i^{*}, g_i^{*}\}$, where

$$\mathbf{z}_i^{*} = T_i^{*}(\mathbf{x}_{\gamma_{1:i}}^{*,i}) = \begin{cases} \mathbf{x}_{\gamma_1}^{*,1}, & \text{if } i = 1, \\ \left[\mathbf{x}_{\gamma_i}^{*,i}, y_{i-1}^{\Delta}\right], & \text{otherwise;} \end{cases} \quad y_i^{*} = \begin{cases} M_1(\mathbf{x}_{\boldsymbol{\alpha}_1}^{*,1}), & \text{if } i = 1, \\ M_i(\mathbf{x}_{\boldsymbol{\alpha}_i}^{*,i}, y_{i-1}^{\Delta}), & \text{otherwise;} \end{cases} \quad g_i^{*} = S_i(\mathbf{x}_{\boldsymbol{\beta}_i}^{*,i}, y_i^{*}) \tag{17}$$

The objective of training intermediate models is to mitigate the propagated prediction uncertainty of the $i$-th GP, rather than to improve the prediction accuracy of themselves. Therefore, it is regarded as passive learning in this work.

### 4.2 *Stopping criteria*

To determine which GP models require active updating during each iteration, we define:

$$\mathrm{EAI}_i = \overline{\mathrm{CoV}}[\hat{P}_{F_i}] + \mathbb{E}_{\mathbf{X}}\left[\sum_{k=1}^{i}\sqrt{\mathrm{VP}_k\left(T_k^{G}(\mathbf{X}_{\gamma_{1:k}})\right)}\right], \tag{18}$$

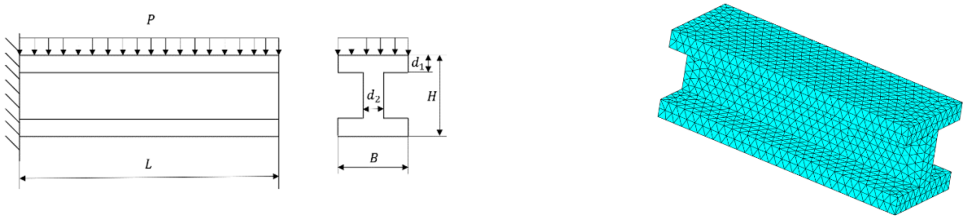
quantifies the total propagated prediction uncertainty up to stage $i$. Only subsystems satisfy $\mathrm{EAI}_i \geq \mathrm{EAI}_{\mathrm{tol}}$ are actively updated and $\mathrm{EAI}_{\mathrm{tol}} = 0.05$ is recommended. When the problem is highly nonlinear, EAI may fail to converge. Therefore, another stopping criterion is formulated as a backstop to ensure convergence of the algorithm

$$\delta_i = \max_i\left(\frac{|\hat{P}_{F_i} - \hat{P}_{F_{i-1}}|}{\hat{P}_{F_{i-1}}}\right) < \delta_{tol}, \qquad i \neq 1. \tag{19}$$

Eq. (19) must be satisfied for 3 consecutive iterations before convergence is accepted and $\delta_{tol} = 0.5\%$ is fixed for the illustrated example. The algorithm is deemed converged when either stopping criterion is met. The compared methods are summarized in Table 1.

## 5 Example: Aluminum profile beam FEM Model

An aluminum profile beam operating in proximity to a heat-dissipation system that periodically emits heated gas is illustrated in Figure 1. Assuming that the interval between emissions is sufficiently long for the beam to return to ambient temperature, the maximum temperature experienced by the beam occurs immediately after the cessation of gas emission at time $t$. The first-stage failure mode is defined as the average temperature of the beam exceeding $T_{max}$, whereas the second-stage failure mode corresponds to the beam deflection exceeding $\gamma_{max}$. For simplicity, the temperature field within the beam after heating by the emitted gas is approximated as spatially uniform, with its value taken as the mean of the original, nonuniform temperature field $T_m$. Using average values for all inputs, we find that the error in the predicted maximum deflection introduced by this uniform-temperature assumption is only 0.1%. The reference temperature in the second FEM model is set to 150 °C.




(a) Illustraion (b) Mesh

**Figure 1.** Aluminum profile bending beam

The subsystem responses and the corresponding performance functions are:

$$\begin{aligned} T_h &= M_1(\mathbf{X}_{\boldsymbol{\alpha}_1}) \qquad g_1 = S_1(\mathbf{X}_{\boldsymbol{\beta}_1}, T_h) = T_{max} - T_h \\ \gamma &= M_2(\mathbf{X}_{\boldsymbol{\alpha}_2}, T_h); \quad g_2 = S_2(\mathbf{X}_{\boldsymbol{\beta}_2}, \gamma) = \gamma_{max} - \gamma, \end{aligned} \tag{20}$$

where $\mathbf{X}_{\boldsymbol{\alpha}_1} = [L, H, B, d_1, d_2, \kappa, \rho, c, T_g, T_0, h, t]$, $\mathbf{X}_{\boldsymbol{\alpha}_2} = [L, H, B, d_1, d_2, \rho, E_0, \nu, \alpha, P]$ and $\mathbf{X}_{\boldsymbol{\beta}_{12}} = \varnothing$ are random inputs. The detailed information of inputs is listed in Table 1.

**Table 1.** Distribution information

| Variables | Distribution | Mean | C.o.V | Variables | Distribution | Mean | C.o.V |
|---|---|---|---|---|---|---|---|
| $L$ / m | Log-normal | 0.3 | 0.01 | $T_0$ / °C | Uniform | 20 | 0.02 |
| $H$ / m | Log-normal | 0.1 | 0.01 | $h$ / (W/(m$^2$.K)) | Uniform | 58 | 0.02 |
| $B$ / m | Log-normal | 0.1 | 0.01 | $t$ / s | Uniform | 150 | 0.05 |
| $d_1$ / m | Log-normal | 0.02 | 0.01 | $E_0$ / GPa | Log-normal | 69 | 0.01 |
| $d_2$ / m | Log-normal | 0.02 | 0.01 | $\nu$ | Log-normal | 0.33 | 0.01 |
| $\kappa$ / (W/m/K) | Uniform | 167 | 0.01 | $\alpha$ / (1/K) | Uniform | $12.7\times10^{-6}$ | 0.01 |
| $\rho$ / (kg/m$^3$) | Uniform | 2700 | 0.01 | $P$ / MPa | Uniform | 4 | 0.05 |
| $c$ / (J/kg/K) | Uniform | 896 | 0.01 | $k$ | Deterministic | $2.5\times10^{-4}$ | – |
| $T_g$ / °C | Uniform | 500 | 0.02 | – | – | – | – |

The compared methods are summarized in Table 2. For the all-in-one model, the response of the $i$-th subsystem is trained using the minimum value among the responses of subsystems 1 to $i$.

**Table 2.** Information of compared methods

| Method | Stopping criterion | Type |
|---|---|---|
| HALPI | See Subsection 4.2 | Hierarchical |
| ALPI (Dang et al. 2021) | $\overline{\mathrm{Cov}}[\hat{P}_F] < 0.1$ | All-in-one |
| AK-MCS (Echard et al. 2011) | $U_{min} > 2$ | All-in-one |
| AK-SM (Constant et al. 2025) | $\hat{\epsilon}_{max} < 0.05$ | All-in-one |

A maximum of 200 model evaluations is allowed for all methods, and ten initial samples are used to construct the starting surrogate models. The averaged results from 20 independent runs are summarized in Table 3.

**Table 3.** Comparison of different methods in 20 independent runs

| Method | $\hat{P}_{F_1}, \hat{P}_{F_2}$ ($\times10^{-3}$) | Relative error | C.o.V | Number of function calls | Average time (s) |
|---|---|---|---|---|---|
| MCS[1] | 7.15, 28.92 | – | 0.0373, 0.0183 | $10^5$, $10^5$ | – |
| HALPI | 7.03, 28.59 | 1.66%, 1.13% | 0.0174, 0.0135 | 30.00, 39.10 | 664.41 |
| ALPI | –, 529.30 | –, ≥100% | –, 0.6782 | 57.50, 57.50 | 471.15 |
| AK-MCS | –, 508.70 | –, ≥100% | –, 0.3509 | 85.30, 85.30 | 856.92 |
| AK-SM | –, 588.92 | –, ≥100% | –, 0.3345 | 21.10, 179.00 | 1019.29 |

As shown in Table 3, all methods except the proposed one exhibit large estimation errors. The corresponding coefficients of variation further indicate that these methods suffer from severe instability. Moreover, the comparison in terms of the number of model evaluations and average wall-clock time demonstrates that the proposed method achieves a more pronounced efficiency advantage when applied to computationally expensive models such as finite element simulations. It is worth noting that existing methods are typically formulated within an all-in-one framework,

[1]The reference result only run once.

which leads to a more challenging modeling task compared to hierarchical approaches, rather than indicating any inherent deficiency of these methods themselves.

## 6 Conclusions

This work proposed a hierarchical active-learning probabilistic integration (HALPI) framework for reliability analysis of first-order sequential series systems. The method constructs individual Gaussian Process (GP) surrogates for each subsystem and explicitly accounts for both the intrinsic prediction uncertainty of the local GP model and the prediction uncertainty propagated from upstream predictions. Based on this formulation, a new acquisition function was developed to guide the active learning process. The resulting active–passive learning mechanism selectively refines only those surrogate models that significantly influence the reliability estimation.

The effectiveness of the proposed framework was demonstrated through an engineering example. The results showed that the proposed method can accurately estimate stage-wise failure probabilities while requiring substantially fewer model evaluations than existing all-in-one surrogate-based approaches. Overall, the proposed framework provides a reliable and computationally efficient tool for reliability analysis of first-order sequential series systems.

Despite its advantages, the framework has limitations. First, it assumes that the transformations between limit states and subsystem responses are invertible, which may not hold for some cases. Second, the present formulation is restricted to first-order sequential series systems that contains only scalar subsystem responses.

These limitations suggest clear directions for future work. Although the present study focuses on scalar linking variables, the proposed framework can be naturally extended to more general sequential series systems involving vector- or field-valued linking variables. Such extensions would enable reliability analysis of complex engineered systems in which each stage transmits high-dimensional physical information, such as process chains. Such developments would further expand the practicality and scalability of sequential reliability analysis for complex engineered systems.

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