# Advanced Directional Importance Sampling Method for Dynamic Reliability Analysis of Linear Structural Systems under Stochastic Non-Gaussian Loading

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Reliability analysis of dynamic structural systems and its implications for structural design have garnered increasing attention. A prominent approach for addressing this challenge involves Monte Carlo simulation and its variants. Sample-based methods prove insensitive to the dimension of the probability integral but may necessitate a substantial number of realizations for small failure probabilities, resulting in time-consuming computations.

Recently, the Directional Importance Sampling (DIS) method was introduced for linear structural systems, showcasing the ability to estimate small failure probabilities (e.g., $10^{-3}$ or less) with a reduced number of dynamic response simulations (e.g., a few hundred). The efficiency of the DIS method hinges on the explicit calculation of dynamic response, capitalizing on the linearity in Gaussian space. However, when the loading is non-Gaussian, which is common in practice, the dynamic response becomes non-Gaussian, and its value in Gaussian space cannot be directly obtained.

Consequently, the existing DIS method is not applicable to non-Gaussian problems.

This contribution introduces an advanced DIS method, named DIS+, specifically designed for efficient dynamic reliability analysis of linear structural systems under stochastic non-Gaussian loading. The non-Gaussian dynamic response is simulated using a unified Hermite polynomial model based on its first four moments, enabling the explicit calculation of its associated Gaussian counterpart. With the dynamic response in hand, the DIS procedure can be easily executed in two steps. Firstly, the failure probability at fixed instants is calculated based on the first four moments of the dynamic response. Subsequently, the direction corresponding to the obtained reliability index is analytically determined. In the second step, a few hundred random direction samples are generated according to the importance sampling density function, and the failure probability is calculated based on the corresponding estimator. Notably, sampling is only required in the second step, and the number of samples required is relatively small. Furthermore, the calculation of the reliability index employs an explicit formula, enhancing the efficiency of the DIS+ method. The application of the proposed DIS+ method is illustrated through a series of examples, showcasing its accuracy and efficiency for dynamic reliability analysis.