

Approximate Bayesian Computations based on fractional-order statistics

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Context

The design of functional components for use in demanding applications is largely founded on numerical approximations of the sets of differential equations that describe the physical processes in our everyday life. In this way, the dynamic and static responses of a complicated structural component to an estimated load can be predicted long before it has been produced. However, some criticism exists with respect to purely virtual deterministic design optimisations using numerical modelling schemes. This criticism finds its root in the various sources of uncertainty that are commonly encountered when designing structural components. Examples of uncertainties in this context include imprecisely defined parameters of the constitutive material models, unclearly defined boundary conditions or physical aspects of the material behaviour that are omitted from the models. These uncertainties need to be explicitly accounted for in the model to ensure its validity [1].

The class of methods belonging to the field of Bayesian Model Updating [2] is particularly powerful in the context of identifying and quantifying relevant uncertainties in those simulation models. Usually, a very high computational cost is associated to these approaches. This is caused by the often-required underlying Markov Chain Monte Carlo simulation schemes. However, recent advances in the field of Approximate Bayesian Computation allow to replace costly likelihood evaluations by a more efficient approach based on difference metrics between selected statistical quantities of the uncertain model quantities and a set of measured data. The correct choice of the correct statistical difference metric is however strongly non-trivial. Fractional moments [3] might provide a good solution to this issue since one fractional moment can carry information on multiple integer statistical moments, effectively compressing the data, and hence, reducing the calculation cost.

Objectives

The purpose of this thesis is to develop a computational framework to include information provided by fractional moments in to an Approximate Bayesian Model Updating framework. Specific goals are:

- Implement an effective framework to estimate fractional moments of uncertain model responses based on defined uncertainties
- Design a statistical metric to describe the difference between the fractional moments of a set of measurement data and the results of the numerical model that is subjected to uncertainty
- Plug the framework and metric into an Approximate Bayesian Computation framework to obtain a highly efficient Bayesian Model Updating framework.

Required Skills

To develop this project, the following skills are considered a plus.

- Strong mathematical background.
- Knowledge of software for numerical analysis (e.g. Matlab / Python / Julia) and simulation (e.g. Abaqus).
- Reading and writing skills in English.

Application

In case that you are interested in this project, please follow these steps.

1. Read the associated bibliography (see below).
2. Prepare a short motivation letter addressing the following issues:
 - a. Your interest in developing this project.
 - b. The reasons that make you a good candidate for developing this project.
 - c. Intended dates for working in the project.
3. Send the motivation letter to the supervisors via E-mail and ask for an exploratory meeting.

Bibliography

[1] Concepts of Model V&V, Los Alamos report LA-14167-MS

[2] A. Lye, A. Cicirello, and E. Patelli, ‘Sampling methods for solving Bayesian model updating problems: A tutorial’, *Mechanical Systems and Signal Processing*, vol. 159, p. 107760, Oct. 2021, doi: [10.1016/j.ymssp.2021.107760](https://doi.org/10.1016/j.ymssp.2021.107760).

[3] C. Ding, C. Dang, M. A. Valdebenito, M. G. R. Faes, M. Broggi, and M. Beer, ‘First-passage probability estimation of high-dimensional nonlinear stochastic dynamic systems by a fractional moments-based mixture distribution approach’, *Mechanical Systems and Signal Processing*, vol. 185, p. 109775, Feb. 2023, doi: [10.1016/j.ymssp.2022.109775](https://doi.org/10.1016/j.ymssp.2022.109775).