Analytical solutions to the Whittle-Matern Kernel

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Context

Many of the loads on engineering components, structures, and systems, as well as the constitutive properties of these assets exhibit a stochastic nature. This assertion is based on the observation that these quantities exhibit apparent variability in time and/or space. In this context, the theoretical framework of random fields has proven an excellent means for capturing inherent uncertainty [1]. Typically, the auto-correlation of such stochastic process is governed by a pre-defined auto-correlation function (also often referred to as `kernel'). This function describes the correlation between two random variables in the stochastic process as a function of the distance in time/space between them. Recent developments by the CRE showed that especially the Whittle-Mattern 3/2 and 5/2 kernels exhibit appealing features for the modelling of random fields, but also in the more general context of building machine learning models [2].

The generation of samples from such random field depends strongly on the availability of the eigen-functions and -values of this kernel. While numerical schemes based on, e.g., Galerkin schemes, exist, the availability of analytical solutions is often preferable for computational reasons. So far, only the Matern ½ and Matern 3/2 kernels have available analytical solutions, while the derivation of these solutions for the 5/2 kernel is still an open question.

Objectives

This thesis aims at developing analytical solutions or approximations for the eigenfunctions of a Whittle-Matern 5/2 kernel, which can then be used in the analysis of random fields in engineering systems. Specific objectives include:

- Understanding the routes which have been followed to derive the analytical solutions of the WM ½ and WM 3/2 kernels
- The derivation of analytical solutions for the eigenfunctions of the WM 5/2 kernel
- Application of these solutions to an engineering case where some key properties are subjected to random fields

Required Skills

To develop this project, the following skills are considered a plus.

- Strong analytical and mathematical background.
- Knowledge of software for numerical analysis (e.g. Matlab / Python / Julia) and simulation (e.g. Abaqus).
- Reading and writing skills in English.

Application

In case that you are interested in this project, please follow these steps.

1. Read the associated bibliography (see below).
2. Prepare a short motivation letter addressing the following issues:
    a. Your interest in developing this project.
    b. The reasons that make you a good candidate for developing this project.
    c. Intended dates for working in the project.
3. Send the motivation letter to the supervisors via E-mail and ask for an exploratory meeting.

Bibliography

[1] W. Betz, I. Papaioannou, and D. Straub, 'Numerical methods for the discretization of random fields by means of the Karhunen-Loève expansion', *Comput. Methods Appl. Mech. Eng.*, vol. 271, pp. 109–129, 2014.

[2] M. G. R. Faes, M. Broggi, P. D. Spanos, and M. Beer, 'Elucidating appealing features of differentiable auto-correlation functions: A study on the modified exponential kernel', *Probabilistic Engineering Mechanics*, vol. 69, p. 103269, Jul. 2022.