A machine learning approach for efficient and robust resistance spot welding monitoring

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Abstract

The estimation of the weld nugget diameter generated by the resistance spot welding process is a crucial element in the assessment of the overall quality of the weld and plays a major role in in-line process control. The process is crucial to produce end-products in many industries such as aviation, aerospace, automotive and other industrial areas. A modern car body contains typically several thousands of welds produced by resistance spot welding, setting an ideal scene for in-line process control. Current state of the art monitoring methods are based on several features extracted from the dynamic resistance signal. However, the accuracy of those is generally not high. In this work, a method for predicting the nugget diameter based on the combination of unsupervised deep learning and Gaussian process regression is developed. Autoencoders are adopted to extract features from the dynamic resistance curve in a low dimensional representation. These features embodies underlying information on the process, possibly unobservable or not detectable by any other currently existing approach. Next, a Gaussian process regression model is trained to link those features to the target weld nugget diameter. Compared with the currently popular geometrical attributes approach, the results show that the model has a higher prediction accuracy in nugget diameter prediction, whilst remaining a low cost implementation in an industrial setting. These results are supported by several cases, derived directly from common industrial bottlenecks. Both cases indicate a strong potential with the new AE-GPR approach, with consistently improved results compared to the currently popular geometrical attributes approach.

Keywords: Resistance spot welding, Nugget diameter, deep learning, machine learning

1 Introduction

Resistance spot welding (RSW) is a highly efficient, low cost and easy realisable joining technique. The technique has been used extensively
and is crucial to produce end-products in many

industries such as aviation, aerospace, automotive and other industrial areas. The process employs two welding electrodes that press two or more overlapping sheet-like workpieces together. The heat generated by the then-applied electric current, in correspondence with Joule's law, causes

local melting at the workpiece's common faying 63 12 surface, leading to joining these workpieces. This 64 13 ease of operation contributed vastly to the quality 65 14 and automation of the production of modern car 15 bodies, which contain typically several thousands 67 16 of welds produced by RSW. The safety and reli-17 68 ability of current automobile industry is only one 18 of many examples that notably profited from the 19 valuable RSW process. 20

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To a large extent, this success can be 72 21 attributed to the ease of automation of the RSW 22 process in an assembly line. Further, RSW also 23 combines high strength joining with production 24 flexibility, low cost and fast throughput. Thanks 76 25 to its wide-spread use, RSW has grown to a 77 26 mature joining technique, with literature dating 78 27 back to the second half of the 19th century (exper-28 imentally oriented) [8] and early 2000's (on-line 29 monitoring, Finite Element methods) [12, 13]. 81 30 Yet, in industrial practice, it still suffers from a 31 high sensitivity to often uncontrollable and vari-32 able process conditions. This makes the RSW 33 process extremely vulnerable to environmental 85 34 effects, surface conditions, misalignment, wear, 86 35 etc. Inevitably, abnormal welding conditions dra-36 matically reduce the consistency of welds, which 37 generally leads to significant degradation of the 38 weld quality. Therefore, it is a vital task to con-39 trol and monitor the quality of the welding process 40 [10]. Also, in order to increase productivity and 41 achieve a robust final assembly, an attempt to 42 minimise the number of required spot welds is 94 43 made. This is only possible when consistent and 95 44 sufficient weld quality can be guaranteed [23, 32]. 45 For the latter, a common technique remains non-46 destructive testing, based on a random subset of 47 the workpieces on the production site. However, 99 48 these weld quality estimations can only be exam- 100 49 ined off-line, making it impossible to receive brisk 101 50 and pertinent information. Furthermore, it is very 102 51 cost inefficient for mass production environments, 103 52 where RSW is vastly present. 104 53

In the context of process monitoring, real-time 105 54 weld quality estimation based on data-driven tech- 106 55 niques are becoming ever more common [3, 11, 21]. 107 56 These approaches typically link process parame- 108 57 ters and on-line measurements to product quality 109 58 metrics in order to guard the process. In this 110 59 respect, machine learning approaches yield very 111 60 fast black-box models enabling on-line application 112 61 for process control. However, multiple problems 62

arise with these black-box models: (a) These models are known to have only limited value when extrapolation is required. Their use is most relevant for well-confined and -controlled processes that enable the generation of a clear, industrially representative and comprehensive data set for the model training. There are large discrepancies between a lab-environment or numerically-made data-set and an industrial data-set, which is prone to inaccuracy due to changing variables or boundary conditions that come with the large change in environment. (b) Quantitative measurement capabilities for the process response are limited in the RSW process, hence eliminating the industrial applicability of data-hungry algorithms such as most supervised deep-learning toolboxes [20].

Various in-line measurement techniques for the RSW process are investigated in literature, and can be classified based on the quantity of measurement (e.g., force, current, time) and their corresponding measurement device. Some of these techniques show promising results regarding inline prediction of the weld nugget diameter, which is usually the primary choice for the Quality Indicator (QI) of the process [4]. A first class of prediction models makes use of mechanical measurements, e.g., displacements [24], forces [31] or acoustic emission [7]. While these are possible sources of valuable process information, and state of the art technology for measurement of the required quantities is proven achievable on industrial scale, the reliability, accuracy and flexibility of these models remain a challenge [25], leading to their limited use in an industrial context. A second class of techniques focuses on monitoring through electrical signals. In this context, dynamic resistance (DR) measurements are widely investigated and implemented in industrial practice [9, 22, 29]. There are several milestones in the progress of monitoring the dynamic resistance. In 2002, Cho and Rhee [5] calculated dynamic resistance based on current and voltage from the primary part of the transformer. Further breakthroughs include quality estimators by means of a Hopfield network, presented in [6], Artificial Neural Networks (ANN) [17], welding quality classifiers by means of Probabilistic Neural Networks (PNN) [27] and a random forest model based on features of the dynamic resistance curves [25]. Measurement of dynamic resistance has become





152 Fig. 1: Theoretical dynamic resistance curve interpretation and characterisation. Top images illustrate the evolution of the weld nugget at the given phase and are made by a phantom VEO 640 ultra high speed camera. Based on the original graph of D.W. Dickinson et al. [8] 158

160 the accepted paradigm in industry [1, 15, 30], 113 161 and has been implemented in several commercially 114 162 available power sources as a quality monitoring 115 163 and evaluation tool. For this reason, it is selected 116 as primary feature in this work. Furthermore, there 117 165 is a clear link between the evolution of the weld 118 166 nugget and the dynamic resistance, as illustrated 119 167 in figure 1. For more detail on the subject, the 120 168 reader is referred to [8]. 121 169

1.1 Motivation 122

While extensive literature exists regarding the 172 123 online monitoring and the quality assessment 173 124 of the spot welding process based on dynamic 174 125 resistance and alternative measurements, many 175 126 challenges remain. Many of the aforementioned 176 127 techniques suffer from drawbacks that hinder their 177 128 optimal cost-effective and fully automated appli-¹⁷⁸ 129 cation for RSW processes in industrial practice. 179 130 These shortcomings are the following. 180 131

181 1. Current dynamic resistance based techniques 132 fail to establish an accurate prediction when 182 133 variations on the input signal are present, 183 134 either due to process parameter alterations or 184 135 inherent randomness in the process. 185 136

2. The dynamic resistance curve is containing information that is not necessarily observable in a time signal, and therefore lost by currently existing techniques. Consequently, the widely investigated techniques based on the geometric attributes of the dynamic resistance measurement (see section 3.2) are not fully exploiting their potential.

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3. Alternatives for the measurement of dynamic resistance are lacking robustness, mainly due to the type of measurement. This leads to either economical or infrastructural burdens, which render them less interesting for industrial application.

In an effort to remedy these shortcomings, this paper investigates a weld quality monitoring approach based on the underlying parametric dependencies of the dynamic resistance during the RSW process. The weld nugget diameter serves as the main driving Quality Indicator (QI) for this research. The capability of predicting the weld nugget diameter by means of limited measurements is an important aspect of the developed approach. This is important to keep the number of man-hours required to measure the data-set feasible. Furthermore, the technique should be able to cope with variations, re-calibrations or other possible variability within industrial application.

We propose deep learning autoencoders to discover a low dimensional representation that captures the underlying causes of the resistance in the secondary circuit of the welding machine during the RSW process. This allows for a sparse representation that can be leveraged towards inline prediction of the weld nugget diameter. The method is demonstrated on an experimental dataset to clarify the technique and to compare it with the geometrical attributes approach (elaborated in section 3), by means of computational performance, prediction accuracy, advantages and disadvantages.

Next, the method is demonstrated on several example problems, showing that the technique has large potential on diverse problems.

The paper is structured as follows:

• Section 2 elaborates on the conducted experiments, as well as the required hardware both for experiments, measurements and processing the data.

- Section 3 discusses the geometrical attributes ²³⁴ approach, which serves as a reference for the ²³⁵ novel methods developed in the next sections, ²³⁶
- Section 4 introduces the autoencoder based 237
 approach, 238
- Section 5 applies the introduced method to two ²³⁹
 case studies to illustrate its application and ²⁴⁰
 performance, ²⁴¹
- Section 6 lists the most important conclusions 242
 of this manuscript. 243

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¹⁹⁶ 2 Experimental setup

Experiments described in this work are per-197 formed on an ARO servo-driven RSW machine 198 with a 1000 Hz, 90 kVA DC power source, as 199 depicted in figure 2. The machine is equipped 200 with water cooled electrode caps with an ISO 201 5821:2009 FE-15.8-5.5-30 geometry. Three exper-202 iments were conducted using process parameters 203 as summarised in table 1. For each experiment, 204 the complete set of process parameters and the 205 number of realisations n is provided. Experimen-206 tal set 1 represents a limited run in a production 207 setup under ideal conditions, where variations are 208 only generated due to randomness in the process 209 and systematic uncertainty due to measurement 210 accuracy. Experimental set 2 is a set of multiple 211 experiments, with each subset having a different 212 value for a selected process parameter. For this 213 work, the current is adjusted, which is one of the 214 most influential process parameters of the pro-215 cess [8]. The rationale behind this case study is 216 to prove the flexibility of the technique over a 217 wide range of machine parameters. Experimen-218 tal set 3 is a set of multiple experiments, with 219 each subset having a geometrical adjustment of 220 the electrode clamp, causing a variation in the ²⁴⁷ 221 248 static electrode bulk resistance (EBR), which is 222 part of the secondary circuit. This is realised by ²⁴⁹ 223 adding custom build raisers (shims) between the ²⁵⁰ 224 251 electrode and its holder. Due to the principle 225 252 of stacking multiple thin shims, the conductivity 226 decreases significantly, causing the resistivity to $^{\scriptscriptstyle 253}$ 22 254 increase with only a minor increase in extra mate-228 rial required. For the three examined cases, the $^{\scriptscriptstyle 255}$ 229 256 resistance between two predetermined points, one 230 above the added shims and one below the added ²⁵⁷ 231 shims, is measured to be 11.9, 108.4 and 371.5 $\mu\Omega^{-258}$ 232 259 respectively. This case is included to illustrate the 233 260

performance of the technique for a common problem in an industrial setting, where the change of welding electrodes or welding clamps inherently causes a variation in the overall resistance of the machine's electrical circuit. The welded specimen are low carbon steel samples of 20 by 70 mm and thickness of 1 mm and are welded in as-delivered condition. Data is acquired by a Dewetron DEWE2-a4L data acquisition system at a sampling rate of 2 MHz. Acquired signals include (1) electrical voltage over the welding electrodes and (2) electrical current in the welding circuit, measured by a PEM RFT 300S Rogowski coil and preamplifier.



Fig. 2: Setup of the machine *(left)*, with location of shims for experimental set 3 *(right)*, top-down: zero, 3 and 5 shims.

All samples are labelled by physical measurements of the weld nugget diameters according to ISO 10447:2015 (specifies the procedures and recommended tooling to be used for peel and chisel testing of resistance spot and projection welds. ISO 10447:2015 applies to welds made in two or more sheets in the thickness range of 0,5 mm to 3,0 mm). The process parameters are chosen such that the nominal nugget diameters correspond to the welding lobe diagram according to ISO 14327:2004, $3.5\sqrt{t}$, with t the thickness of a single plate. Furthermore, the weld time and force differ greatly between the conducted experiments. This is deliberately determined as such, to provide 296
a demonstration on a case where the weld is gen-297
erated in a short time window as well as a case 298
where a weld nugget is formed slower. 299

²⁶⁵ 3 Dynamic resistance based ²⁶⁶ monitoring

This section describes, discusses and illustrates 304
the geometrical attributes regression model as pre-305
sented in literature. It serves as a reference for the 306
developments in the remainder of this work. 307

The geometrical attributes regression model 271 is currently the most commonly adopted pre-272 diction model in industry. The model is based 273 on input-output pairs, respectively from on-line 274 measurements and nugget diameter measurements 275 stemming from destructive testing (as elaborated 276 in section 2). Figure 3 visualises the main flow of 277 the approach. It consists of k welds being gener-278 ated and measured by means of an experimental 279 campaign. The samples are *peeled*, a destructive 280 testing method (ISO 10447:2015) for determin-281 ing the diameter of the weld nugget. When the 282 demanded samples k are generated, the data is 283 subjected to a training algorithm based on the 284 input-output pairs. All additional samples (i > k)285 are then predicted based on the trained model. 286

²⁸⁷ 3.1 Feature extraction

Measurement of dynamic resistance is one of the most effective techniques for quality monitoring and estimation, aided by the fact that measurements are straightforward, including electrical current and voltage in the secondary circuit. Next, the dynamic resistance signal is obtained according to Ohm's law, i.e.

$$R(t) = \frac{U(t)}{I(t)},\tag{1}$$

with R(t) the dynamic resistance, U(t) the the voltage, I(t) the welding current and t the welding time.

During the process, the welding machine forms a closed circuit with the secondary circuit of a transformer, ensuring a solid mechanical assembly between tooling and work pieces. The closed circuit is modelled in terms of their individual resistances. In this resistance model, the electrical resistances of the transformer, the mechanical assembly, and the work pieces are represented as respectively R_t, R_m and R_l . The resistances R_t and R_m are assumed constant during the process. The resistance of the work pieces is split into three components:

1. the bulk resistance of the sheet metal (R_b) ,

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- 2. the interface resistance between electrodes and sheet metal (R_c) ,
- 3. the contact resistance of the parts surfaces (R_f) .

For the general case of two pieces of sheet metal, assumed equal in properties and dimensions, the resistance of the work pieces is:

$$R_l = 2R_b + 2R_c + R_f.$$
 (2)

As illustrated in figure 1, there are multiple stages during the process, causing a large fluctuation of R_l . The geometrical attributes approach,



Fig. 3: Workflow a. General prediction model structure for RSW

No.	Current (kA)	Force (kN)	Time (ms)	EBR	Ν
1	7.2	5	210	cte	174
2	6.0 - 6.6 - 7.2	2.5	70	cte	50 - 50 - 50
3	7.6	2.5	80	var.	50 - 50 - 50

Table 1: Overview of conducted experiments and their process parameters used in this paper. EBR: Electrode bulk resistance setup, n: amount of welds

elaborated in section 3.2, refers to several points 311 that characterise the dynamical resistance curve. 312 Extracted points from the curve are based on e.g., 313 peaks and slopes. Figure 4 illustrates several key 314 points of the curve, respectively the initial peak 315 R_0 in phase 1, the pit R_{α} in phase 2, the peak at 316 the beginning of phase 4, also commonly known 317 as the beta peak R_{β} and the last value of the 318 DR curve R_{γ} , also respectively the times $t_0 t_{\alpha}$, 319 t_{β} and t_{γ} . Next, several critical derivatives are 320 selected, the mean value R_m , the slopes m_1 and 321 m_2 and the resistance variance $dR1 = R_\beta - R_\alpha$ 322 337 and $dR2 = R_{\gamma} - R_{\beta}$ 323



Fig. 4: Theoretical dynamic resistance curve, with selected features for the geometrical attributes approach, adopted from [9].

However, one drawback of this technique is 324 that these measured parameters fluctuate heavily 325 during a single weld due to the time-varying cur-326 rent generated by a mid frequency direct current 327 (MFDC) power supply. The dynamic resistance, 328 derived according to Eq. 1, is subjected to a sig-329 nal filter in order to obtain the main trend. Zhang 330 et al. [26] evaluated the raw signals and acknowl-331 edged, based on Fourier spectrum analysis, that 332 periodic features are key to the large fluctuations. 333 They applied a fourth-order digital low pass fil-334 ter with a cut-off frequency of 50 Hz. For this 335

 Table 2: Overview of correlations between derived
 features and the QI, based on experimental case 1

feature (f_i)	%	feature (f_i)	%
t_0	1.68	R_0	3.06
t_{lpha}	-31.37	R_{α}	33.36
t_{eta}	-30.60	R_{β}	20.62
t_{γ}	-20.43	R_{γ}	-8.56
m_1	8.60	m_2	6.30
dR_1	-4.45	dR_2	49.73
R_m	44.29		

study, the concept of the moving average filtering is selected to eliminate the interference of periodic signals effectively [33].

3.2 Geometrical attributes model 339

Post-filtering the dynamic resistance signal provides a noise-free curve where the features described in section 3.1 (Figure 4) are observable. Table 2 gives an overview of the correlations between the features according to figure 4 and the QI, based on case 1 from table 1. This table shows that correlations between inputs and the QI are present. This confirms the applicability of the geometrical attributes approach, as discussed by [9]. The ensemble of these, or similarly derived, correlations, superseded with generic regression analysis, are current state of the art methods for RSW quality indication. This is visualised as the left part in figure 5. Part A, the feature selection, yields the aforementioned features of the dynamic resistance curve. Next, in part B, a multiple linear regression model describes the relationship between the features and the QI. The selection of features is case dependent, and relates to the most significant features of the curve, with a minimum of six points, without significantly affecting the model performance [9]. The regression model is described as

$$q = \beta_0 + \beta_1 f_1 + \beta_2 f_2 + \dots + \beta_k f_k + \varepsilon \quad (3)$$

with q the QI response, f the regressor variable, β the regression coefficients and ε the error term. This overall workflow makes it possible to project new data-points onto the regression model and predict (interpolate) an estimate for the QI.



Fig. 5: Workflow b. Technical

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³⁴⁵ 4 Autoencoder based weld ³⁴⁶ quality monitoring

This section discusses the main innovation of this ³⁶⁹ study, replacing both part A and part B of the ³⁷⁰ technique elaborated in section 3.2, as also illus- ³⁷¹ trated in the right part of figure 5. In the following, ³⁷² the method is introduced and discussed, as well as ³⁷³ illustrated based on experimental set 1 (table 1). ³⁷⁴

353 4.1 Part A: Feature extraction

³⁵⁴ 4.1.1 The curse of dimensionality

Part A in this work focuses on improving the ³⁷⁹ amount of information that is extracted from the ³⁸⁰ dynamic resistance curve. For this, an efficient ³⁸¹ coding is required that is capable of learning a ³⁸² low dimensional representation for a set of data. ³⁸³ Opposed to the method of manually selecting ³⁸⁴ points, recent advances in deep learning methodologies are proven to be very efficient in gathering interesting features in the data, which are possibly unobservable or not detectable to the engineer during manual evaluation of the data. However, especially for this application, this poses specific challenges regarding the architecture of the network. Indeed, with a sampling rate of 2Mhz and the process yielding > 200ms of welding time, a very high amount of data is gathered on multiple channels, which serves as the input of the network. The main problem is that the computational cost of the neural network scales exponentially with the data, by cause of a connection that is required to each neuron in the next layer, according to:

$$z_j^l = \varphi(v_j) = \varphi\left(\sum_{i=0}^k w_i . z_i^{l-1}\right), \qquad (4)$$

where z_j^l is the value z for neuron number j in layer l, φ represents the activation function, usually a sigmoid function ranging from -1 to 1, $\varphi(v_i) =$ $\frac{2}{(1+exp(-2.v_j))} - 1, z_i^{l-1}$ representing neuron number i from layer l-1, w_i the weight assigned to each connection with the previous layer, k the number of neurons in layer l-1 and $z_0 = \pm 1$ for adding a bias $b = w_0$ to the summation operator, yielding v_i . Evaluating this key equation is computationally not a large effort. However, due to the architecture of neural networks, it has to be solved numerous times during training. As such the total training effort is increased drastically. This is often referred to as the curse of dimensionality, referring to problems that occur when dealing with data in high-dimensional spaces. It prevents strategies to work efficiently, while creating problems concerning computational expenses, which do not occur in low-dimensional spaces.

Multiple techniques for dimensionality reduction exist. They can be divided into convex and non-convex techniques, where convex techniques optimise an objective function that does not contain any local optima, e.g., Principal Component Analysis (PCA), Kernel PCA, Isomaps, Local Linear Embedding (LLE) and non-convex techniques optimise objective functions that do contain local optima, e.g., Locally Linear Coordination (LLC), manifold charting or autoencoders [14, 28]. For a competent model, capable of being deployed in an on-line context, a marginal computation cost is



Fig. 6: Autoencoder topology

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envisaged for the projection of a series of points 411
to the low dimensional space, but also the ability 412
to embed new high-dimensional data points into 413
an existing low-dimensional data representation is 414
important. For these reasons, autoencoders have 415
been selected for the dimension reduction in this 416
work. 417

4.1.2 Autoencoder based dimension reduction

Autoencoders, a type of artificial neural network, are the proposed solution for this work, based on their efficiency, non-linear transformation and intuitive nature [2].

These feed-forward networks have an odd number of hidden layers hL_i , with $i = 1 \dots n_l$ and n_l the amount of layers, where the hidden layers are dimensioned such that the layer in the middle has a lower amount of neurons than the first and last layer. This separates the autoencoder in an input layer, an encoder part, the middle layer with $d \leq D$, a decoder part and the reconstructed layer:

$$\mathbf{y}^{\mathbf{D}} \xrightarrow{Encoder} \mathbf{y}^{\mathbf{d}} \xrightarrow{Decoder} \mathbf{y}'^{\mathbf{D}},$$
 (5)

where $\mathbf{y}^{\mathbf{D}}$ is the measured data, D the number of 399 time-steps in the input data, and d the amount 400 of neurons in the middle layer. The objective of 401 the autoencoder is to generate this neural network 402 architecture such that $\mathbf{y'^D} \approx \mathbf{y^D}$. The autoen-403 coder is an unsupervised learning technique, since 404 its goal is to minimise an error in reconstructing 405 $\mathbf{y}^{\mathbf{D}}$. The input layer $\mathbf{y}^{\mathbf{D}}$ has D neurons, where each 406 neuron represents an individual parameter from 407 the dataset. This data is reconstructed in the final 408 layer $\mathbf{y'}^{\mathbf{D}}$ of equal dimension D, as illustrated in 409 figure 6. 410

The centre layer in the network $\mathbf{y}^{\mathbf{d}}$ represents the original data in a lower dimension d, while preserving as much structure as possible from the dataset $\mathbf{y}^{\mathbf{D}}$. The resulting low dimensional representation in this centre layer functions as the input for further processing, which has the benefit of working with far less data without losing essential information.

Mapping from the input vector to another vector by means of an encoder, based on the general equation of a neural network topology (eq 4) gives:

$$y^d = \varphi(W^1 y^D + b^1) \tag{6}$$

and for the reconstruction through a decoder:

$$y'^D = \varphi(W^2 y^d + b^2). \tag{7}$$

The network is trained by minimising a loss function, which includes regularisation terms. Apart from the mean squared error function, an L2 regularisation term $\lambda * \Omega_{weights}$ and sparsity regularisation term $\beta * \Omega_{sparsity}$ are added to the loss function. The L2 regularisation term forces the weights to remain small, by adding a penalty to the loss function when weights are increasing. The sparsity regularisation term attempts to enforce a constraint on the sparsity of the output from the hidden layer. The cost function for training the autoencoder based on N samples yields

$$\mathcal{L}(y^{D}, y'^{D}) = \frac{1}{N} \sum_{n=1}^{N} \sum_{j=1}^{D} \left(y_{jn}^{D} - y'_{jn}^{D} \right)^{2} + \lambda * \Omega_{weights} + \beta * \Omega_{sparsity} \quad (8)$$

422 with λ the coefficient for the L2 regulariza-423 tion term and β the coefficient for the sparsity 424 regularization term.

The workflow for applying this metric for the dynamic resistance curve is illustrated in figure 7.



Fig. 7: Workflow c. Autoencoder training principle

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In this figure, there is a clear distinction 446 427 between the data for training and testing. It also 447 428 indicates the main workflow, where eq. 8 is used 448 429 to train the reconstruction of the measured signal 449 430 and to evaluate the reconstruction of test data. 450 431 Furthermore, the figure illustrates that part B is 451 432 connected to the low dimensional layer, in the 452 433 centre of the autoencoder. 453 434

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435 4.1.3 Illustration

The autoencoder based dimension reduction is
now illustrated based on experimental case 1 from
table 1. Out of 174 experiments, 90 % serves
as training data, whereas the remaining 10 % is
test-data to evaluate the performance.

Figure 8 illustrates the performance of the $^{462}_{462}$ network for one sample from y^D_{Test} , where one $^{462}_{463}$ dynamic resistance curve is plotted next to its $^{464}_{465}$ reconstructed counterpart. Since the error is $^{465}_{465}$ nearly negligible, the instantaneous reconstruction $^{466}_{465}$

error ϵ_{AE} is given, defined as:

$$\epsilon_{AE} = \frac{y'^D - y^D}{y^D}.100. \tag{9}$$

In this example, the signal is compressed from D = 450.000 data-points into a middle layer of the autoencoder, represented by y^d , where d = 15. The hyper-parameters for the network are summarised in table 3.



Fig. 8: Measured and reconstructed signal with means of an autoencoder network, with topology 450k - 15 - 450k

Note that the reconstruction is only required for training of the autoencoder. For the purpose of dimensionality reduction, the encoder projection, resulting in the reduced space y^d is an important step to come to an efficient regression model, as illustrated in figure 7.

At this point, it can be concluded that the presented method is capable of projecting the measured dataset into a reduced space y^d , which acts as a low dimensional space. The reconstructed projected data, by means of the autoencoder, performs approximately equal to the measured data, as the error is nearly negligible. Furthermore, the projected data contains nearly all information to reconstruct the data in a low dimensional space, thus possessing at least as valuable information as the manually determined points, as described in section 3.2. Therefore, the reduced space y^d can serve as an input to current the QI prediction step using generalised techniques, e.g., multiple regression. It should be noted that the projected data has no physical meaning in the process, opposed to the selected features from section 3.2. The added benefit of this method is the robustness of

 Table 3: Hyperparameters for training Autoencoder

Encoder-TF	Sigmoid
Decoder-TF	Sigmoid
d	15
L2 weight coef. $[\lambda]$	0.009
Sparsity Proportion $[\hat{\rho}_i]$	0.719
Sparsity Regularization $[\beta]$	1.08
Normalised data	Yes

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512 the algorithm, which is unlikely to suffer abnor-470 513 malites, e.g, where expulsion could cause an effect 471 in the curve, resulting in misjudgement or wrong 472 515 interpretation of the data. Additionally, the mea-473 516 sured signal is prone to various effects like time 474 517 shift, originated during the processing of the data, 475 caused by the required filtering techniques of the 476 signal. The presented approach is also insensitive 477 to these effects. 478

479 4.2 Part B: QI prediction

Part B of the novel prediction model (see figure 5) 480 requires an algorithm capable of giving a robust 481 regression between input-output, respectively the 482 measured signals during the welding process pro-483 jected on a reduced space \mathcal{Z} by means of an 484 autoencoder, and the measured nugget diameter 485 519 of the weld (QI). Due to the low amount of avail-486 520 able labelled data, training a neural network based 487 521 on input-output pairs tends to be troublesome, 488 522 in particular overfitting is a main concern. This 489 523 should not be confused with the neural network on 490 524 which the autoencoder from section 4.1 is based, 491 525 as the latter is an unsupervised technique, which 492 526 does not require labelled data. 493 527

494 4.2.1 Gaussian Process Regression

A powerful tool, ideal for this problem is Gaussian 530 495 Process Regression (GPR). In it's original form, 531 496 Gaussian Process modelling is a statistical inter- 532 497 polation method that exploits Gaussian processes 533 498 to interpolate a series of complex functions. The 534 499 technique works well on small datasets, and has 535 500 the capability to provide uncertainty metrics on 536 501 the predictions. Gaussian process modelling, also 537 502 known as Kriging, was introduced in the context 538 503 of meta-modelling in the works of Sacks et al. [19], 539 504 in which the original form of Kriging, as developed 540 505 in the Master's thesis of D. Krige [16], served as a 506 backbone to represent an input/output mapping 507

of an expensive computational model. For application in machine learning, Kriging has evolved as both regression and classification tool, and has proven to be a treasured algorithm [18].

The required dataset \mathcal{D} with N observation is presented as

$$\mathcal{D} = \left\{ \left(y_i^d, q_i \right) | i = 1, \dots, N \right\}$$
(10)

with y_i^d the vector with multiple input variables and q_i as a measure for the QI, which is continuous, as this is a regression case.

In essence, GPR serves as an interpolation model for a regression problem mapping the QI on the variables.

They key equation of GPR is presented as:

$$q = \mathcal{M}(y) + \varepsilon, \tag{11}$$

with $\mathcal{M}(x)$ the GP model. It is assumed that the observed responses are noisy and the noise ε follows a zero-mean Gaussian distribution

$$\varepsilon \sim \mathcal{N}(0, \sum_{n}),$$
 (12)

where \sum_{n} is the covariance matrix of the noise term. For this work, the noise variance is identified as general heteroscedastic, in which the noise can differ for each observed response. For a more in depth study of GPR, the reader is referred to the citations in previous section.

An important feature of this type of model is the applicability of K-fold cross-validation. The main idea of this is to re-evaluate the model with a subset of the training data. This allows to optimise the hyper-parameters of the model, based on evaluating the model with each subset of data, thus minimising a cross-validation error. This yields a model, capable of working under limited data, and providing efficient predictors with minimal effort, unlikely to suffer from overfitting. Performancewise, training this model takes significantly longer than linear regression. However, as elaborated in the introduction, the computational requirement for training is irrelevant for an in-line paradigm. What is of interest is the required time to evaluate a single sample. This will be studied for the illustrative data set in the next section.



Fig. 9: Predicted vs. actual QI based on the AE-GPR approach *(right)* compared to the Geometrical attributes approach *(left)* for the reference case

541 4.2.2 Illustration

The methodology is presented on the data result-542 ing from section 4.1.3. Computation experiments 543 and the training of the proposed algorithms are 544 performed on a server, consisting of 2 AMD Epyc 545 7601 32-core CPU's, 512 GB memory and an 546 NVIDIA Tesla V100 - 32 GB graphical accelera-547 tor. For benchmarking computational effort of the 548 proposed algorithm, the system is limited to only 549 1 core, while GPU acceleration is disabled. 550

Measured D

569 The summary of computational costs for both 551 570 the reference and the newly developed approach 552 57.1 are given in table 4. Comparison of the required 553 time for the regression approaches clearly reveals 554 that the GPR technique is more expensive. This 555 574 is due to the fact that substantially more time 556 575 is required to determine the optimal hyperparam-557 576 eters, compared to the key, and only equation 558 577 required for multiple linear regression 3. Apart 559 578 from that, the table also shows that especially the 560 579 training of the unsupervised deep learning based 561 dimension reduction technique into an autoen-562 coder (AE) necessitates a high performance com-563 putational device. However, the cost for training is 564 583 only a small investment for a continuously guarded 565 584 process. Table 4 also summarises the computa-566 585 tional costs for training and propagating a single 567

Table 4: Computational costs for the framework, *AE*: Autoencoder based dimension reduction, *GPR*: Gaussian Process Regression

Measured D

		Training	1 Sample
Part A	Geom. attributes	$1.8 \ge 10^3 s$	0.46 s
	AE	$5 \ge 10^4 s$	$0.83 \ s$
Part B	Linear regression	1.01 s	$0.03 \ s$
	GPR	91 s	$0.04 \ s$

sample. This shows that propagating a single experiment through the full AE-GPR framework requires only a marginally higher cost compared to the approach based on the geometrical attributes.

In order to assess the performance of the developed method, figure 9 illustrates the correlation between the predicted QI and the measured QI, both for the geometrical attributes approach (left) and the novel AE-GPR approach (right).

A clear improvement over the geometrical approach is noticeable. Not only does the Root Mean Squared Error (RMSE) improve vastly, from 0.26 to 0.18, it is also clearly visible that the geometrical approach fails to predict the overall nugget diameter. Specifically, the graph based on the geometrical approach shows a horizontal scatter, indicating a larger spread on the measured QI, but a small spread on the predicted

QI. Furthermore, in addition to the continuous 633 586 mean regressor estimate by the GPR model, also 634 587 confidence bounds are provided, based on the 635 588 available training data and known variance of the 636 589 GPR model. For this case, the 99 % bounds are 637 590 provided, showing a narrow band encapsulating 638 591 the data. As a final verification, the algorithm 639 592 is subjected to several testing points, which were 640 593 excluded from the training. The latter is rep- 641 594 resented by 10 % of the available data and is 642 595 visualised by the red crosses. The RMSE for the 643 596 test points improves from 0.21 to 0.13 for the test 644 597 data, which emphasises the performance of the 645 598 model by resulting in a prediction well within the 646 599 bounds predicted by the algorithm. 600 647

$_{601}$ 5 Verification of the proposed $_{649}^{649}$

652 To benchmark the accuracy and robustness of the 603 653 introduced approach, the algorithm is subjected to 604 654 two more cases. Compared to case 1 of section 3.2605 655 for which an experimental set with fixed process 606 656 parameters was used (thus only yielding process 60 657 variability on both input and output), the next 608 658 two cases include highly relevant industrial events 609 representing common process variations. 610

⁶¹¹ 5.1 Current variation

661 This case (table 1, No.2) is a dataset combin-612 662 ing data from three consecutive experimental runs 613 663 with respectively altering the current as described 614 664 in table 1. The goal of this case is to determine 615 665 the robustness of the AE-GPR approach based on 616 666 a broader set of input signals, as could be the 617 667 case in an industrial setup where low- and flexible 618 668 production is key and in-line monitoring is envis-619 669 aged. First, applying a dimension reduction on the 620 670 measured signals reduces the dataset from 450.000 621 671 samples to 15, while only losing < 1 % of infor-622 672 mation, based on mean-squared error estimates. 623 673 This is illustrated in figure 11, where the origi-624 674 nal curve, the reconstructed curve and the error 625 675 are illustrated. Analysing the measured and pre-626 676 dicted data, illustrated in figure 10, clearly reveals 627 677 several meaningful results. 628

First, no separate data clusters can be distinguished according to the difference in welding currents. Yet, the three datasets are separated by colour, which reveals the difference in nugget diameter in function of process parameters. Second, the measured results with 6.0kA and 7.2kAhave a wider spread of data compared to the set welded with 6.6kA. Third, advancing to the results of the prediction model, there is a clear improvement of the AE-GPR method over the geometrical approach. The RMSE is improved, from 0.39 to 0.13. It is clearly visible that the predictions based on the AE-GPR approach are more confined than the predictions based on the geometrical approach. Furthermore, the predictor based on the geometrical approach fails clearly in determining the measured nugget diameter on the samples stemming from the subset yielding the smallest nugget diameter. The provided 99 % confidence bounds show a narrow band encapsulating the data.

As a final verification, the algorithm is subjected to several testing points which were excluded from the training. The latter is represented by 10 % of the available data and is visualised by the red crosses. The RMSE for the test points improves from 0.62 to 0.13 for testing data. Both the RMSE and graphical representation of the predicted value indicate a prediction well within the predicted bounds of the algorithm.

5.2 EBR variation

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For the final example, a highly relevant industrial case is elected. One of the advantages of many state-of-the-art welding controllers is their ability to monitor themself and intervene during a single weld. This is required to achieve a certain threshold of total current flow and is often realised by extending the weld time or increasing the current. An important reason why this threshold is often not achieved is the variation in the closed loop resistance in the secondary circuit. These variations affect the welding process and potentially the reaction of the controller, causing a variation in the weld nugget diameter. Also, it is well known within RSW industry that the weld nugget diameter is negatively affected by variation in the closed loop resistance in the secondary circuit. The latter is often a consequence of machine disturbances, e.g., maintenance, re-torquing or swapping tools. Thus re-calibration is required when adjustments are made to the welding heads to counter this



Fig. 10: Predicted vs. actual QI based on the AE-GPR approach *(right)* compared to the Geometrical attributes approach *(left)* for the current variation case

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Fig. 11: Measured and reconstructed signal with means of an autoencoder network, with topology 450k - 15 - 450k for the current variation case

effect. The goal of this third case is to demon-⁷⁰⁷ strate the potential of our approach when the ⁷⁰⁸ aforementioned effects are present.⁷⁰⁹

710 The dataset is a combination of three consec-683 utive experimental runs altering the resistance of 711 68 712 the secondary circuit by means of custom shims 685 713 between the welding head and electrode holder, 686 714 while the compensation of the controller is dis-687 715 abled. The resistance between two fixed points in 688 the mechanical assembly is respectively measured 716 689 717 to be 11.9, 108.4 and 371.5 $\mu\Omega$. This is achieved 690 by respectively zero, 3 and 5 shims, as illustrated 691

in figure ??. This variation is a constant resistance during the process and is part of R_m , as elaborated in section 3.1.

Analysing the measured and predicted data, illustrated in figure 12, clearly reveals several meaningful results. First, the influence of the change from 11.9 to $108.4\mu\Omega$ has an almost negligible effect on the measured nugget diameter with respectively an average of 4.24 and 4.09 mm and equal standard deviation of 0.18 mm. The experiments where the resistance is increased to $371.5\mu\Omega$ indicate a noticeably smaller nugget diameter, with an average of 3.18 mm and standard deviation of 0.33 mm. Second, applying the proposed dimension reduction by means of a trained autoencoder reduces the dataset from 140.000 samples to 30, while only losing < 1% of information, based on mean-squared error estimates.

Third, advancing to the results of the prediction model, there is a clear improvement over the geometrical approach. For the set with 11.9 and 108.4 $\mu\Omega$, the variation in nugget diameter is rather low, yet the AE-GPR is capable of predicting the nugget diameter more accurate compared to the geometrical approach. The nugget diameters for the set with 371.5 $\mu\Omega$ range from 2.7



Fig. 12: Predicted vs. actual QI based on the AE-GPR approach *(right)* compared to the Geometrical attributes approach *(left)* for the EBR variation case

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to 4.1 mm, while the geometrical approach predicts only between 3.0 and 3.5 mm. The AE-GPR 744
appraoch predicts this set, both for the training 745
and test points. The RMSE of the predictions 746
improves clearly, from 0.23 to 0.15. For this case, 747
the 99% bounds are provided, showing a narrow 748
band encapsulating the data. 749

The test points are represented by 10 % of 750 the available data. The RMSE for the test points 751 improves from 0.18 to 0.15, resulting in a pre- 752 diction well within the predicted bounds of the 753 algorithm. 754

730 6 Conclusion

This paper presents a novel approach for an effec- 758 731 tive predictor for the nugget diameter, i.e., the 759 732 main quality indicator (QI) of a resistance spot 760 733 welding process, based on on-line measured pro-761 734 cess data. The most important feature of the 762 735 developed methodology is the combination of 763 736 deep learning for dimension reduction, and the 764 737 consequent machine learning prediction tool. We 765 738 demonstrate how to use unsupervised deep learn- 766 739 ing in the form of an autoencoder to discover 767 740 a low-dimensional transformation in which the 768 741 parameters characterise a pattern that embodies 742

underlying information on the process, possibly unobservable or not detectable by any other currently existing approach. The underlying information is transformed into a low dimensional space, which is an ideal scene for a Gaussian process regression model linking the input data through the autoencoder to the measured nugget diameter. The model is trained on a limited set of data, leading to a low cost implementation in an industrial setting. The technique is presented in an example case which clearly indicates that it leads to an improved QI prediction compared to the state of the art geometrical attributes approach. The technique is presented on two additional cases, each pinpointing a specific bottleneck within industry related to the RSW process. Both cases are analysed and indicate very promising results, where the new AE-GPR approach has consistently improved results over the geometrical approach. It should be noted that the time to train the proposed model is substantially higher. However, the investment cost is rather low compared to the benefit in an in-line monitoring system. We conclude that the proposed method can be readily applied to an in-line context for quality assessment in industrial RSW applications, as the time for evaluating in an in-line context is suit- 811
able, and accuracy is vastly improved over existing 812
techniques. 813

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