First excursion probability of dynamical systems: A review on computational methods

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Highlights

- We provide a synthesis of the literature on the first excursion probability of dynamical systems and their recent developments.
- We discussed the efficacy of existing methods in addressing various challenges associated with first excursion problems in complex dynamic systems.
- We present a synthesis of the most recent methods proposed in the field of reliability and evaluate their potential for addressing complex first excursion problems.

Abstract

The theory of dynamic reliability, predicated on the first excursion failure criterion, holds significant importance in the domains of seismic and wind resistance of structures, as well as in the assessment of the reliability of machinery and airplanes. This theoretical framework offers a mathematical description of failure probabilities, which serve as critical indicators for the safety evaluations of dynamic systems. However, dynamical systems such as large structures, machines or airplanes are composed of numerous members and nodes that may be influenced by uncertainties related to loads, geometric imperfections, and material properties. The inherent high-dimensional randomness and pronounced nonlinear coupling effects contribute to the complexity and implicit nature of the system failure modes in these systems. Consequently, the computation of the first excursion probability for complex dynamical systems presents a formidable challenge that necessitates comprehensive investigation. To summarize the current methodologies, this paper delineates a state-of-the-art review of dynamic reliability theory, with a particular emphasis on its potential to address the first excursion probability in dynamical systems.

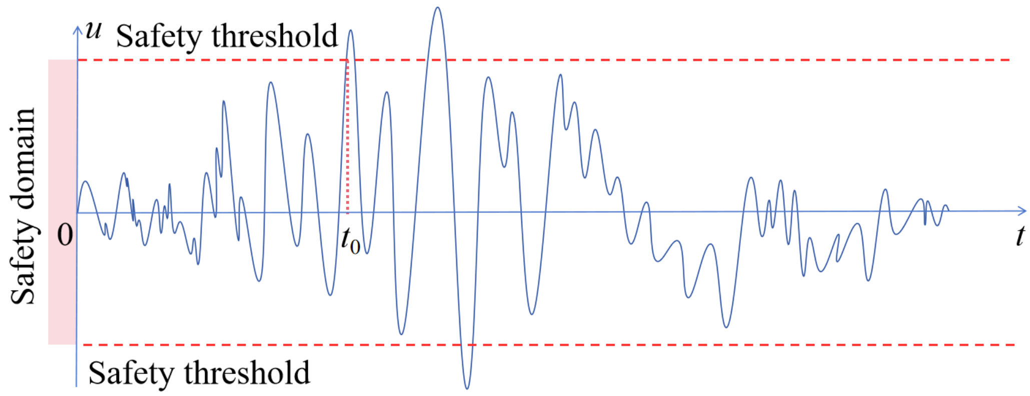
Keywords: Reliability theory, First excursion probability, Dynamical systems, Random excitation, Random vibrations

Introduction

In the processes of designing and assessing structural systems, natural hazards, such as wind loads and earthquakes, can lead to the partial failure or collapse of structures, resulting in catastrophic damage. Similar phenomena are observed when dealing with mechanical engineering systems such as vibration induced by imbalances in rotating machines or turbulence loads on aircraft wings. Therefore, it is essential to evaluate the reliability performance of dynamical systems under stochastic loads to mitigate potential safety risks. While some existing studies utilize random variables to model extreme random processes, they are limited in their ability to analyze the transient reliability. Consequently, the dynamic reliability analysis of system responses to random excitation will become increasingly critical in the future [1].

Dynamic reliability refers to the probability that a system will successfully perform its intended function when subjected to random excitations within specified time frames and conditions. The first excursion failure criterion assesses whether a system has lost its performance by comparing physical quantities associated with failure to a critical value or safety threshold. This criterion provides a simplified and idealized model, as illustrated in Fig. 1. By determining whether any of the random outputs $u(t)$ exceed a safety threshold level b within a specified time duration T, the first excursion probability P_f can be expressed as:

$$P_f = p\left(\exists t \in [0,T] : |u(t)| > b\right) \tag{1}$$



t_0 denoted the first excursion time

Fig. 1 The First excursion problem of random processes.

During the preliminary phases of investigating the first excursion probability, the emphasis was placed on examining the random process and its correlation with thresholds. In 1945, Rice formulated a set of infinite series expansions and complex multiple integral equations to address the issue of exceeding fixed boundaries in random processes. This seminal work established a foundational framework for dynamic reliability analysis rooted in the theory of random processes [2]. Subsequently, Middleton validated Rice's formula for calculating the first excursion probability and further investigated the challenges posed by random processes exceeding safety thresholds [3]. Their contributions have significantly advanced the theoretical research pertaining to the first excursion problem. Consequently, the Poisson process method, predicated on the cross assumption, has been extensively utilized in addressing the first excursion problem[4].

Research concerning the first excursion probability of nonlinear systems commenced in the 1970s, predominantly focusing on systems characterized by small damping, weak nonlinearity, or bilinear restoring forces. The stochastic response of dynamic systems can be solved by solving the Fokker-Planck-Kolmogorov (FPK) equation and its variations, or using equivalent linearization methods, which are still widely used in structural first excursion probability analysis[5].

Subsequent investigations have demonstrated that the first excursion probability under specified thresholds can be equivalently derived through the relevant extreme value distribution [6]. This facilitates the transformation of the time-dependent dynamic reliability problem into a corresponding time-invariant problem. Over the past two decades, probability density evolution methods have also made significant progress, providing a unified framework for dynamic and static reliability analysis [7].

In addition to semi-analytical methods, numerical simulation techniques have been employed to determine the first excursion probability. Due to the high computational demands of Monte Carlo simulation for extremely small probability analysis [8], several efficient sampling methods [9] and surrogate model methods[10] have been proposed, yielding effective solutions for the first excursion problem of complex dynamical systems.

When conducting the first excursion probability analysis in practical engineering systems, numerous challenges persist. These challenges include the small failure probability problem in high-dimensional nonlinear systems, the twofold random uncertainty challenge associated with uncertainty of structural physical-geometrical parameters and the randomness of external excitation, as well as the dynamic reliability concerns associated with complex systems subjected to multiple failure modes. To address the potential challenges that may arise in the first excursion probability analysis of dynamic systems, this paper presents the development of the first excursion probability calculation method and its subsequent application in resolving these challenges. In this paper, existing methods are categorized into two main categories: semi-analytical methods and numerical simulation methods. For each category, an overview and summary of each method are presented, along with a discussion of the latest advancements. Furthermore, this paper addresses the existing gaps in current methodologies and suggests potential avenues for future research. The organization of the paper is illustrated in Fig. 2.

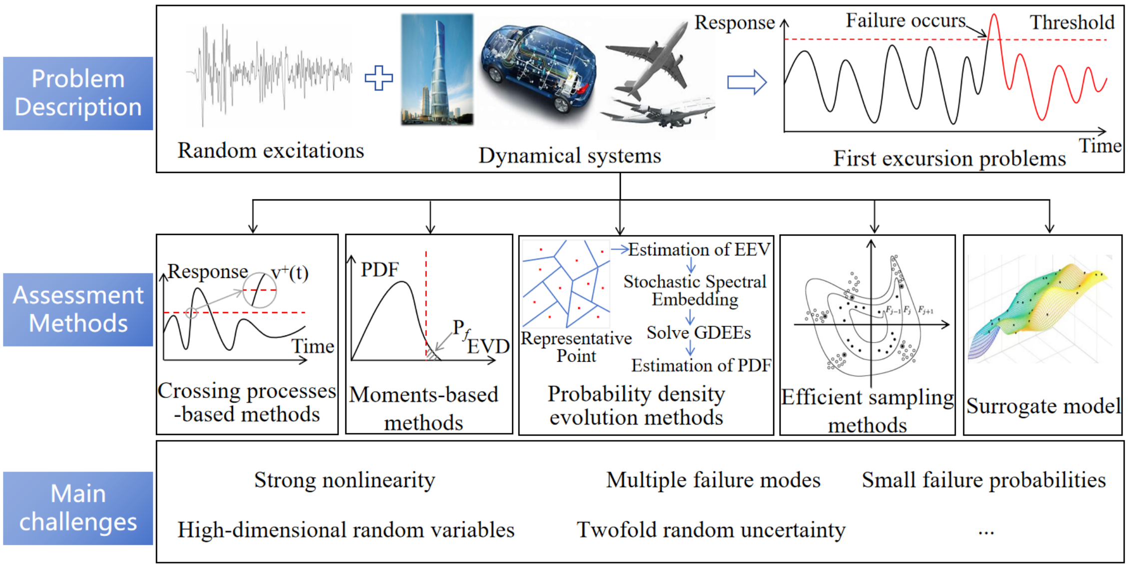


Fig. 2 The organization of the paper.

2 Semi-analytical Methods

2.1 Crossing processes-based methods

Assuming that the events of structural response exceeding the safety threshold are independent, it can be hypothesized that the crossing times follow a Poisson distribution. Based on this assumption, Coleman [11] proposed the well-known Poisson process-based method and derived analytical solutions for the first excursion probabilities. Within a given duration *T*, the probability of *u*(*t*) crossing the threshold *b* in *i* times can be expressed as:

$$p\{n(T)=i\}=\frac{1}{i!}\left[\int_0^T v_b(\tau)d\tau\right]^i \exp\left[-\int_0^T v_b(\tau)d\tau\right] \tag{2}$$

where $v_b(\tau)$ represents the crossing rate of the response *u*(*t*) surpassing the threshold *b*, which expressed as:

$$v_b=\frac{1}{2\pi}\frac{\sigma_{\dot{u}}}{\sigma_u}\exp\left(-\frac{b^2}{2\sigma_u^2}\right) \tag{3}$$

where σ_u represents the standard deviation of displacement and $\sigma_{\dot{u}}$ represents the standard deviation of velocity.

For events characterized by high safety thresholds, the assumption of a Poisson process is often deemed acceptable due to the rarity of exceeding these thresholds. However, when the structural response is akin to a narrowband process defined by slowly varying sine waves, the occurrence of a crossing event in one cycle significantly increases the likelihood of another crossing event in the subsequent cycle. This behavior indicates that crossing events tend to exhibit clustering, thereby rendering the assumption of a Poisson process inappropriate [12]. To enhance the precision of first excursion probability evaluations, Corotis et al. [13] advocate for the consideration of crossing events as a Markov process, wherein the probability of the next crossing event is contingent solely upon the current event. Yang and Shinozuka [14][15] employed the point process method alongside the maximum entropy principle to investigate the first excursion problem in normally distributed narrowband responses. The main formula is as follows:

$$p(T) = 1 - [2(\lambda_2 / \pi)^{1/2} \exp(-\lambda_1^2) / \mathrm{erfc}(\lambda_1)] \quad (4)$$

where $p(T)$ represents the first excursion probability in [0,T]. In Eq. (4), $\mathrm{erfc}(\lambda_1)$ is the complementary error function,and λ_1 and $\lambda_2$can be obtained from the following equations:

$$\mu_2 / \mu_1^2 = (\gamma - \lambda_1)^{-2} / 2 - \lambda_1 / (\gamma - \lambda_1) \quad (5)$$

$$\lambda_2 \mu_2 = 0.5 + \lambda_1^2 - \gamma \lambda_1 \quad (6)$$

μ_1 and μ_2 represents the mean value and the second moment of total number of excursions in [0,T], and

$$\gamma = \exp(-\lambda_1^2) / [\sqrt{\pi} \mathrm{erfc}(\lambda_1)] \quad (7)$$

Their findings suggest that the point process approach produces several useful approximations for the first excursion probability, particularly those based on the concepts of the Markov process, the clump-size and the non-approaching random points. Subsequent research has introduced various models for crossing events, including the classical Vanmarcke approximation model [12][16],and the stochastic process envelope model [17][18],among others.

Based on the assumption of a Poisson process, a critical step in determining failure probability is the analysis of the out-crossing rate. Consequently, the calculation of the out-crossing rate between system responses and safety limits in complex scenarios has emerged as a significant research objective. For instance, the PHI2 method [19] expresses the out-crossing rate through a classical two-component parallel system reliability model, which transforms time-variant reliability into time-invariant reliability at successive time instants. Naess [20] employed the random process spectral decomposition technique to develop a differential calculation method for the out-crossing rate of Gaussian process exceedance within the framework of the second-order Volterra series. He [21] employed Simpson's numerical integration rule to approximate the first three spectral moments of the linear system responses subjected to non-stationary random excitation. Li et al. [22] derived closed-form analytic solutions for calculating the average out-crossing rate and first excursion probability of a non-stationary log-normal process.Cai et al.[23] proposed a new analytical formula to determine the mean outcrossing rate of nonstationary non-Gaussian performance functions. These methodologies provide a quantitative framework for calculating the average out-crossing rate of the system in the context of complex excitations, thereby facilitating the evaluation of the first excursion probability of dynamic systems across various complex scenarios.

However, when addressing first excursion problems associated with non-Gaussian excitations or nonlinear systems, it is inappropriate to model the system response directly using Poisson processes. Consequently, approximate methods must be employed for calculations[24]. This limitation constrains the applicability of crossing process-based methods in the first excursion probability analysis of complex dynamic systems.

2.1.1 Latest developments of crossing processes-based methods

Recent research has increasingly focused on the clustering of crossing events in scenarios characterized by stochastic processes exhibiting high autocorrelation and low thresholds. Yi et al. [25]developed a Poisson Branching Process model to effectively capture the statistical dependencies among crossings, thereby facilitating a more precise estimation of the first excursion probability for stationary Gaussian processes. Similarly, Yang et al. [26]introduced the equiprobable joint Gaussian method to mitigate the issue of redundant counting of identical crossing events. They employed the conditional up-crossing rate to compute the first excursion probability by utilizing collectively exhaustive events and the law of total probability, thus addressing the dynamic reliability challenges associated with high-dimensional nonlinear systems.

2.1.2 Summary of crossing processes-based methods

The crossing processes-based methods represent a preliminary approach to addressing the first excursion problem, providing an analytical formula for calculating the first excursion probability. Although these methods can produce a failure probability that approximates the true value, they rely on particular assumptions regarding the crossing event. Consequently, while these methods constitute a significant contribution to the first excursion problem, there remains substantial potential for further development in addressing the challenges encountered in complex dynamic systems.

2.2 Diffusion processes-based methods

The stochastic vibration equation of a nonlinear system subjected to white noise or filtered white noise can be reformulated as an Itô stochastic differential equation. It can be demonstrated that the solution to the Itô equation constitutes a Markov process. The transition probability density function, which comprehensively characterizes the statistical properties of the Markov process, adheres to the

Fokker-Planck-Kolmogorov equation (FPK equation). Consequently, the random response of the structure can be derived by solving the FPK equation. In theory, the FPK equation method constitutes the most rigorous framework for the analysis of nonlinear random vibrations, yielding precise solutions to nonlinear problems. However, solving the FPK equation presents substantial challenges. Even in scenarios involving stationary responses, only a limited number of specific systems permit the determination of exact solutions.

By solving the partial differential equation governing the system's response, such as the Kolmogorov backward equation derived from diffusion theory, it becomes feasible to evaluate the first excursion probability[27]. Assuming that the system state X_t evolves according to the stochastic differential equation, the Kolmogorov backward equation can be expressed as:

$$\frac{\partial p(x,t)}{\partial t}=\mu(x,t)\frac{\partial p(x,t)}{\partial x}+\frac{1}{2}\sigma^2(x,t)\frac{\partial^2 p(x,t)}{\partial x^2} \tag{8}$$

where $p(x,t)$ represents the probability of the random process being in state x, and t represents time. In Eq. (8), $\mu(x,t)\frac{\partial p(x,t)}{\partial x}$ represents the rate of state transition and $\mu(x,t)$ is called the drift term; $\frac{1}{2}\sigma^2(x,t)\frac{\partial^2 p(x,t)}{\partial x^2}$ represents the diffusion rate and $\sigma^2(x,t)$ is called the diffusion term.

Under specific conditions, the dynamic response of a nonlinear system subjected to non-white noise random excitation may be approximated by a Markov diffusion process. The drift and diffusion coefficients of the FPK equation associated with this approximate diffusion process can be derived from the motion equations of the given system, expressed as:

$$\dot{A}=K_1(A,t)+K_2(A,t)\eta(t) \tag{9}$$

$$K_1(A,t)=-\frac{1}{2}\varphi(A)A(t)+\frac{\pi S[\omega(A),t]}{2A(t)\omega^2(A)} \tag{10}$$

$$K_2(A,t) = \frac{\{\pi S[\omega(A),t]\}^{1/2}}{\omega(A)} \tag{11}$$

where the response amplitude envelope A is a slowly varying function with respect to time and can therefore be treated as a constant over one cycle of oscillation, which approximately governs the temporal evolution of the amplitude $A(t)$; $\eta(t)$ is a zero mean and delta correlated Gaussian process of intensity 1, i.e., $E[\eta(t)]=0$; and $E[\eta(t)\eta(t+\tau)]=\delta(\tau)$, with $\delta(\tau)$ being the Dirac delta function. In Eq. (10) and Eq. (11), $\varphi(A)$ and $\omega^2(A)$ represent the equivalent damping element and the natural frequency of the system, respectively, while $S[\omega(A),t]$ represents the evolutionary non-separable power spectrum of the form. The Fokker-Planck (F-P) equation, which corresponds to stochastic differential equations, is cast in the form:

$$\frac{\partial}{\partial t} p(A,t|A',t') = -\frac{\partial}{\partial A}[K_1(A,t)p(A,t|A',t')] + \frac{1}{2}\frac{\partial^2}{\partial A^2}[K_2^2(A,t)p(A,t|A',t')] \tag{12}$$

The coefficients of this approximate diffusion process equation can be derived through the appropriate application of random averaging to the motion equation of the given system, and the solution of the averaged FPK equation can yield an approximate response of the original system[28]. This approach is particularly effective for analyzing the stochastic response of lightly damped nonlinear SDOF systems subjected to broadband random excitation.Additionally, approximate analysis techniques based on random averaging and statistical linearization can effectively address various nonlinear and lagging systems, as well as diverse forms of random excitation, while maintaining low computational costs [29]. Building upon this foundation, Mitseas et al. [30][31] developed a stochastic incremental dynamic analysis method for calculating the first excursion probability, which is applicable to nonlinear structural systems influenced by random seismic excitation, as well as lagged fractional order structural systems under entirely non-stationary seismic excitation vectors.

The responses PDF of the nonlinear stochastic system excited by white noise is approximated with the exponential function of a polynomial in state variables. The exponential polynomial closure (EPC) method [32] is employed to satisfy the FPK equation in the weak sense of integration with the assumed PDF.In the context of a nonlinear system excited by a discrete Poisson pulse, the probability density function of the system can be characterized through the Kolmogorov-Feller equation. By solving the Kolmogorov-Feller backward equation, the first excursion probability can subsequently be derived [33][34].Based on the aforementioned concept, Zhu et al. [35] investigate nonlinear systems subjected to Poisson white noise excitation by solving the truncated Kolmogorov-Feller equation. Subsequent research has further analyzed the first excursion probability of nonlinear oscillators under combinations of composite noise, utilizing advanced EPC methods [36][37].

2.2.1 Path Integral approach

On the other hand, the Path Integral (PI) approach can also estimate the response PDF and the first excursion probability of low-dimensional nonlinear systems[38]. In essence, the PI approach constitutes a discrete version of the Chapman-Kolmogorov (CK) equation, which is also associated with Markov processes. The fundamental equation of the PI approach can be expressed:

$$p_Z(\mathrm{z},t+\tau)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_Z(z_1,z_2,t+\tau\,|\,\bar{z}_1,\bar{z}_2,t)\,p_Z(\bar{z}_1,\bar{z}_2,t)\,d\bar{z}_1 d\bar{z}_2 \qquad (13)$$

where for compactness $p_Z(\mathrm{z},t+\tau)=p_Z(z_1,z_2,t+\tau)$, and $p_Z(z_1,z_2,t+\tau\,|\,\bar{z}_1,\bar{z}_2,t)$ is the so-called Conditional PDF of the response vector process $Z(t)$. $Z(t)$ is a two-dimensional Markov vector process that satisfies the CK equation, and

$$\dot{Z}(t)=\mathrm{f}(\mathrm{Z},t)+\mathrm{l}V(t) \qquad (14)$$

where $V(t)$ is a modulated white noise process, and $\dot{Z}(t)$ is state variables of nonlinear system, $Z(t)=[Z_1(t)Z_2(t)]^T=[X(t)\dot{X}(t)]^T$, $\mathrm{l}=[0\ 1]^T$ and

$$f(Z,t)=\begin{bmatrix} Z_2 \\ -f(Z_1,Z_2,t) \end{bmatrix}^T \tag{15}$$

The initial PI method has been extensively utilized to assess the response PDF [39][40]and first excursion probability[41][42] of nonlinear systems subject to time-modulated normal white noise excitation. Subsequent investigations have further explored the method considering the case of non-normal excitations, such as Poisson noise [43], Lèvy white noise [44], and composite parameter excitation [45][46].

To address the issue of low computational efficiency associated with PI solutions in high-dimensional MDOF systems, various numerical schemes have been employed to enhance the integration calculation of the CK equation [47][48][49]. Currently, several advanced PI methods have been successfully implemented in the context of the first excursion problem related to practical engineering structures [50][51], thereby demonstrating the potential of these methods in addressing the complex first excursion problem.

2.2.2 Latest developments of diffusion processes-based methods

Recent research has extended existing methods to transient probabilistic analysis of nonlinear systems under complex excitations, including multiplicative and additive modulated stochastic excitations [52], combined harmonic and modulated Gaussian white noise excitations [53], correlated multi-power velocity multiplicative excitations, and additive excitations[54], as well as multiple external and parametric excitations[55].These investigations provide a valuable reference for the first excursion probability analysis of dynamical systems under complex excitation conditions. Guo et al. [56] enhanced the EPC method by integrating time basis functions into the PDF approximation, thereby achieving entirely non-stationary PDF solutions. This implementation facilitates the acquisition of continuous PDF distributions in the time domain, significantly improving computational efficiency without the need for repeated calculations.

The PI method has garnered considerable attention in recent years. For example, the GPU computing-based PI strategy enhances the efficiency of probability response analysis in high-dimensional systems [57][58]. The combination of Laplace integration and PI method can significantly mitigate associated computational costs [59], and has been successfully employed in the first excursion probability analysis of nonlinear SDOF systems that incorporate fractional derivative elements [60], as well as under stationary and time-modulated white noise excitation [61].

Furthermore, the application of artificial neural networks for addressing the FPK equation has garnered significant attention in recent literature [62][63]. Wang et al. [64] introduced a separable Gaussian neural network methodology to derive solutions for the FPK equation within high-dimensional state spaces, successfully obtaining solutions for the high-dimensional FPK equation associated with nonlinear dynamical systems.

2.2.3 Summary of diffusion processes-based methods

In theory, the diffusion process-based methods is the most rigorous tool in nonlinear random vibration analysis, which can determine the exact solution of the first excursion probability of nonlinear dynamical systems. However, solving the FPK equations for complex systems is extremely difficult, and currently there is a relative lack of effective solutions for high-dimensional FPK equations. Therefore, numerous approximation methods grounded in diffusion process theory have been extensively proposed and progressively applied to the first excursion problem associated with complex dynamic systems, indicating a substantial potential for further development in this area.

2.3 Moments-based methods

When the random response of the structure is conceptualized as a sequence of discrete variables, the first excursion probability can be effectively calculated using the corresponding extreme value distribution

(EVD) of the stochastic structural response [65]. Furthermore, by leveraging the EVD, the time-dependent dynamic reliability issue can be converted into a time-invariant equivalent. In this framework, established methodologies for time-invariant structural reliability, such as first-order reliability methods (FORM), second-order reliability methods (SORM) [66], and importance sampling [67], can be directly employed to tackle the first excursion problem.

The EVD of the random structural response can be derived through two methodologies: the analytical approach and the approximate approach. The analytical approach is limited to specific categories of random processes and is generally impractical for addressing the reliability challenges of complex systems. Consequently, the approximate approach to EVD has garnered significant attention within the field. Moments-based methods estimate the first-passage probability by fitting an appropriate parametric distribution model to the EVD, and the free parameters of the distribution model are obtained from the estimated moments of the EVD. The integer moments methods can be adopted to recover the EVD [6][68][69], where high-order integer moments, i.e., skewness and kurtosis, need to be considered. For instance, Low [70] developed a new distribution called the shifted generalized lognormal distribution for fitting four moments. The PDF and CDF of a shifted generalized lognormal distribution variable Y are given by, respectively

$$f_Y(y)=\frac{\alpha}{y-b_1}\exp\left(-\frac{1}{r\varepsilon^r}\left|\ln\frac{y-b_1}{\theta}\right|\right), x>b_1 \tag{16}$$

$$F_Y(y)=\frac{1}{2}+\frac{1}{2}\operatorname{sgn}\left(\frac{y-b_1}{\theta}-1\right)\rho\left(\frac{1}{r},\frac{\left|\frac{\ln\frac{y-b_1}{\theta}}{\varepsilon}\right|^r}{r}\right), x>b_1 \tag{17}$$

where b_1 is the location parameter, θ is the scale parameter, $0<\varepsilon$ and $0<r$

are the shape parameters, the coefficient α is defined as $\alpha = 1/[2r^{1/r}\varepsilon\Gamma(1+1/r)]$, here $\Gamma(\cdot)$ denotes the gamma function, $\mathrm{sgn}(\cdot)$ denotes the signum function, and ρ is the lower incomplete gamma function ratio, i.e., $\rho(s,z) = \int_0^z t^{s-1}e^{-t}dt / \Gamma(s)$.

Based on integer moment method, Zhou et al. [71] combined statistical moment estimation with a displacement generalized normal distribution model, developing a program based on adaptive Bayesian quadrature to evaluate the first four central moments of the equivalent extremum of the structural response. Zhao et al. [72] proposed a fourth-moment Gaussian transformation and derived its inverse transform, which maps the non-Gaussian structural response to a standard Gaussian process, thereby facilitating the evaluation of the first excursion probability.

However, it is difficult to evaluate such high-order integer moments using a small sample size, due to the large variability in their estimators [73]. To alleviate such difficulty, a series of methods based on non-integer moments, such as fractional moments and linear moments, have been developed. These methods include the maximum entropy method [74], the kernel density maximum entropy method [75], and the mixture distribution method [76], among others. Some of these studies provide valuable references for the application of moments-based methods in engineering contexts. For example, Zhang et al. [77] proposed an efficient method for the dynamic reliability assessment of nonlinear structures subjected to non-stationary ground motions utilizing linear moments. Their method for estimating the first four linear moments of extrema of structural responses is developed through the application of the random function-spectral representation model and nonlinear time history analysis, in which principal points of a uniform distribution are incorporated for input estimation. Furthermore, Chen et al. [78] propose a novel methodology for the seismic reliability evaluation of random nonlinear bridges subjected to spatially varying ground motions. The implementation involves the generation of spatially correlated ground motions using a random function-based spectral representation method. Subsequently, the uncertainty of these ground motions is characterized by three fundamental random variables. The first-passage probability of bridges under spatially varying stochastic ground motionsis derived through the equivalent EVD, which is approximated using a flexible four-parameter distribution, specifically the shifted generalized lognormal distribution.

Based on extreme value theory, the EVD estimation method employs probability distribution functions to model extreme value samples, such as the Gumbel distribution [79] and the generalized extreme value distribution [80]. Usually, modeling the main part of the EVD probability distribution is relatively straightforward, while its tail requires in-depth analysis for accurate fitting. In high-dimensional nonlinear problems with small failure probability characteristics, the tail of the fitted EVD probability distribution may exhibit unexpected oscillations, leading to inaccurate evaluations of the first excursion probability. Consequently, it is imperative to pay particular attention to the extreme structural response at the tail of the distribution.

Koo and Der Kiureghian[81] developed an algorithm for identifying the sequence of design points, which determines the tail probability of the nonlinear stochastic response by linearizing a limit-state surface at a designated “design point.” This problem is formulated as a time-invariant reliability problem defined by the limit-state function

$$g(x_0, t_n, \mathrm{v}) = x_0 - X(t_n, \mathrm{v}) \tag{18}$$

With the input stochastic process discretized, the random vibration problem is completely defined by the vectors of random variables v, the former representing system properties that are random, while x_0 and t_n are deterministic parameters in the limit-state function. The design point in the standard normal space is the solution to the constrained optimization problem

$$v^*(x_0, t_n) = \arg\min\{\|v\|^2 : G(x_0, t_n, v) = 0\} \tag{19}$$

This point is defined within the space of standard normal random variables derived from discretizing the input excitation, corresponding to the most probable realization of the excitation that leads to the tail exceedance event. Through this approach, FORM and SORM approximations are implemented at the design point, and can also be used to select sampling distributions in important sampling methods.For small probability problems, the probability content of the failure domain is concentrated around the design point. Therefore, design points are essential in most methods for failure probability estimation [82]. Additionally, Alibrandi and Der Kiureghian [83]developed a gradient-free method for locating the design point in nonlinear stochastic dynamic analysis.

Based on the definition of design point, Fujimura and Der Kiureghian[84] proposed that the tail probability of the linear system is equal to the first-order approximation of the tail probability of the nonlinear system, this property motivating the name Tail Equivalent Linearization Method (TELM). The method employs a discrete representation of the stochastic excitation and concepts from the FORM. For a specified response threshold of the nonlinear system, the equivalent linear system is defined by matching the design points of the linear and nonlinear responses in the space of the standard normal random variables obtained from the discretization of the excitation. The first-order approximation of the tail probability is then given by

$$p[x_0 \le (t_n, v)] = p[G(x_0, t_n, v) \le 0] = \Phi[-\beta(x_0, t_n)] \tag{20}$$

where $\Phi[\cdot]$ is the standard normal cumulative probability function and

$$\beta(x_0, t_n) = \alpha(x_0, t_n) v^*(x_0, t_n) \tag{21}$$

is the reliability index, in which

$$\alpha(x_0,t_n) = -\frac{\nabla_u G(x_0,t_n,\mathrm{v}^*)}{\left\|\nabla_u G(x_0,t_n,\mathrm{v}^*)\right\|} \tag{22}$$

is the normalized negative gradient vector of the limit-state function at the design point. The latter two quantities uniquely define the hyper plane $\beta(x_0,t_n) - \alpha(x_0,t_n)\mathrm{v} = 0$, which is tangent to the limit-state surface at the design point.

Broccardo et al. [85]provided an overview of TELM and discussed its potential advantages. Compared to traditional linearization methods, TELM exhibits greater accuracy in estimating the distribution of tail region responses, rendering it particularly suitable for high-reliability problems. Garrè and Der Kiureghian[86]extended TELM from the time domain to the frequency domain and application to marine structures.Additionally, TELM has been applied to nonlinear mechanical systems subjected to multiple random excitations[87], inelastic multisupport structures subjected to spatially varying stochastic ground motion [88], and structures with degrading materials [89][90]. These studies indicate that TELM can effectively solve the first excursion problem of nonlinear dynamical systems under random excitation.

2.3.1 Latest developments of moments-based methods

At present, determining the EVD of random processes remains an important yet challenging problem in engineering. With the development of reliability methods, new solutions have emerged for the dynamic reliability problem of complex stochastic systems. For example, Chen et al.[91]developed a fractional moments-based mixture distribution approach to estimate the first excursion probability of high-dimensional nonlinear stochastic dynamic systems. Broccardo and Der Kiureghian[92] introduced the Evolution Tail Equivalent Linearization Method, which is based on the approximate algorithm of Priestley's evolution theory, to analyze the transient response of the entire nonlinear stochastic dynamical process. Chen and Lyu [93] developed a numerical method that combines the Chapman-Kolmogorov equation with the Itô stochastic differential equation to evaluate the time-varying probability density function (PDF) of the maximum value of a Markov process or Markov vector process. By constructing an augmented Markov vector process, Lyu et al.[94] calculated the PDF of time-varying extreme value processes in Poisson white noise driven dynamical systems. This method is applicable to high-dimensional nonlinear stochastic dynamical systems and demonstrates calculation accuracy on the order of magnitude of 10^{-4}~10^{-5} at the tail of the cumulative distribution function. Based on these methods, the challenges associated with high-dimensional parameters, strong nonlinearity, and small failure probability in the dynamic reliability analysis of dynamical systems have been effectively addressed.

Additionally, it is noteworthy that some of the latest studies simultaneously consider the uncertainty of structural physical-geometrical parameters and the randomness of external excitation. For instance, Weng et al. [95] proposed a conditional extreme value distribution method to estimate the structural first excursion probability, which incorporates the inherent randomness of structural physical-geometrical parameters as well as external excitations. To extract a small failure probability, the proposed method introduces an intermediate event to represent the realizations of extreme structural responses in the tail of the distribution. Dang et al. [96] improved the mixture distribution method by introducing the moment-generating function to characterize the EVD. This method is capable of accommodating both the randomness arising from structural properties and external seismic excitations, and it is applicable for evaluating multiple EVDs in a single analytical run. Furthermore, it has demonstrated high accuracy in assessing the EVD and small first excursion probability of nonlinear structures with random parameters under stochastic seismic excitations. The findings of these studies indicate that structural random parameters play a significant role in dynamic reliability analysis. Neglecting the randomness associated with structural characteristics may substantially underestimate structural failure probabilities, particularly in high-reliability contexts.

2.3.2 Summary of moments-based methods

The first excursion problem can be reformulated as a probabilistic assessment of the EVD through the computation of high-order moments of the dynamic response. The parameter distribution model-based EVD approximation method has been validated as an effective approach for addressing most nonlinear problems.

On the other hand, TELM based on design point excitation, offers enhanced analytical capabilities for examining the tail of the EVD and addressing the issue of small failure probability in nonlinear systems.Many algorithms, i.e. surrogate modeling, line sampling, subset simulation and cross entropy, are in fact all related to the design point concept.

Recent research has successfully addressed the first excursion problem associated with high-dimensional parameters, strong nonlinearity, and small failure probability by advancing existing time-invariant methods. Relevant algorithms have been developed that consider the uncertainty of structural physical geometric parameters and the randomness of external excitations. While many challenges in complex dynamical systems have been effectively tackled, there remains a relative paucity of research concerning the multiple failure modes of complex dynamical systems based on time-invariant methods.

2.4 Probability density evolution methods

In 2003, Li and Chen [7] developed the Generalized Density Evolution Equation (GDEE) by applying the principle of probability conservation to characterize random events and to decouple the control equations of structural systems. They introduced a Probability Density Evolution Method (PDEM) that is grounded in the fundamental concepts of physical stochastic systems and the principle of probability conservation, thereby providing a unified theoretical framework. This method elucidates the probability density and evolutionary characteristics of stochastic dynamical systems.

Denote the PDF of the random process $u(t)$ as $p(x,t)$.Consider the probability transition process in a fixed domain D with the boundaries ∂D in the state space. The total increment of probability in D over time interval $[t, t+\Delta t]$ yields

$$\Delta P_d = \int_D p(x,t+\Delta t)dx - \int_D p(x,t)dx = \int_D \frac{\partial p(x,t)}{\partial t} dx \Delta t + o(\Delta t) \tag{23}$$

where $o(\Delta t)$ is an infinitesimal of higher order.By deriving the corresponding generalized probability density evolution equation and solving the GDEE, one can obtain the probability information regarding the system's response. Consequently, the probability density evolution method facilitates the determination of the probability density function for the structural response under random excitation.

Within the framework of probability density evolution theory, two primary categories of methods are employed for dynamic reliability evaluation: the absorption boundary process method and the extreme value distribution method. The absorption boundary process method incorporates absorption boundary processes associated with the first excursion failure criterion into the generalized density evolution equation. By solving the probability density function of the structure within the safety domain, this method calculates the dynamic reliability of the structure through integration [97]. Conversely, the extreme value distribution method converts the dynamic reliability of the structure into a one-dimensional integration problem involving the probability density function of the extreme value of the dynamic response. This transformation is achieved by constructing an equivalent extreme value (EEV) event of the structural dynamic response over a specified time period and subsequently solving it [98].

In comparison to sampling methods, the probability density evolution method exhibits superior efficiency and accuracy in addressing the dynamic reliability of simple systems [8]. Most practical engineering problems are characterized by high-dimensional parameters and pervasive nonlinearity. Although the analysis of the GDEE within probability density evolution methods poses significant challenges, numerical methods can be employed to facilitate its solution [99]. Consequently, the implementation of the probability density evolution method in practical scenarios generally encompasses two critical steps: the partitioning of the probability space and the numerical solution of the GDEE.

In the domain of probability space segmentation and the selection of representative points, Li et al. have developed a range of methodologies. These include the mapping dimensionality reduction method [100], the sphere of contact method [101], the number theoretical method [102], the new discrepancy method [103], the partition of the probability-assigned space method [104], and the GF-discrepancy method [105].

The Generalized Density Evolution Equation (GDEE) for stochastic dynamical systems bears resemblance to the conservation equation in fluid mechanics. Consequently, it can be characterized as a first-order hyperbolic partial differential equation with time-varying coefficients, which presents a significant challenge for direct numerical solution techniques. The initial approach to solving the GDEE utilized the finite difference method, specifically employing the Lax-Wendroff scheme. However, this method fails to ensure the preservation of non-negativity in the probability density function. This inadequacy necessitated the integration of a flux limiter to amend the Lax-Wendroff scheme and ascertain that the differential scheme retains the Total Variation Diminishing (TVD) property. Shi et al. [106] developed a non-uniform time step TVD scheme founded on non-uniform grid partitioning technology, thereby introducing an improved probability density evolution equation TVD scheme. The spatial and temporal discretization of the differential method is also constrained by the Courant-Friedrichs-Lewy condition, a challenge addressed by the

Petrov-Galerkin finite element approach to the GDEE proposed by Papadopoulos and Kaloggeris [107]. Furthermore, the δ function sequence solution method introduced by Fan and Li offers an innovative numerical solution for the generalized density evolution equation [108][109].

To mitigate potential issues related to grid sensitivity that may arise during the GDEE solution, Tao and Li [110] proposed a novel difference-wavelet method. This method enhances the finite difference results through the application of nonlinear wavelet density estimation. By determining the optimal scale at each time step, this approach yields more accurate probability density outcomes. Additionally, they developed an integrated evolution method for solving the probability density evolution equation, which incorporates the standard deviation information from each probability subdomain into the probability density evolution equation. Subsequently, they employed a point selection technique based on the Gauss-Frobenius difference, alongside a finite difference format characterized by diminishing total variation, to solve the updated GDEE [111].

Li et al. [112] conducted a study on the application of the probability density evolution method in the dynamic reliability analysis of nonlinear systems. Their findings indicate that the response probability density of random nonlinear structures exhibits typical evolutionary characteristics. However, as time progresses, the evolution process becomes increasingly complex, resulting in significant random fluctuations in the response of the nonlinear system. Although certain enhancement algorithms can address general nonlinear problems [113], challenges remain when confronting high-dimensional complex nonlinear systems.

In order to better handle the first excursion problem of complex dynamical systems, Chen and Yang [114] developed the Direct Probability Integration Method (DPIM) to characterize the propagation of randomness. This method formulates a probability density integral equation based on the principle of probability conservation for stochastic events. By utilizing techniques for partitioning probability space and applying smoothing methods with Dirac's δ function, DPIM effectively addresses the integral equation to derive the probability density function of the system's random response. Through the partitioning of the input probability space and the application of the smoothing technique, the formulations of DPIM are expressed as follows:

$$p(x,t) \approx \sum_{q=1}^{N}\left\{\delta[x-g(\theta_q,t)]\int_{\Omega_{\Theta,q}} p_{\Theta}(\theta)d\theta\right\} \approx \sum_{q=1}^{N}\left\{\frac{1}{\sqrt{2\pi}\upsilon}e^{-[x-g(\theta_q,t)]^2/2\upsilon^2}P_q\right\} \tag{24}$$

where θ_q indicates the q-th representative point in probability space; $\Omega_{\Theta,q}$ denotes the representative region occupied by the q-th representative point; N represents the total number of representative points; υ is the smoothing parameter, specifically the standard deviation of Gaussian distribution; P_q means assigned probability of the q-th representative point.

The generalized density evolution equation and the probability density integral equation of dynamical systems represent differential and integral formulations of the principle of conservation of probability. These equations exhibit an equivalent relationship and are capable of effectively addressing composite random vibration problems that involve both random parameters and random excitations. Furthermore, the numerical solution of the probability density integral equation does not require adherence to the Courant-Friedrichs-Lewy conditions pertaining to time and spatial discretization, which results in significant advantages in computational accuracy and efficiency within dynamic systems [115]. Additionally, the DPIM is applicable to both linear and nonlinear static mechanics and dynamics problems [116][117], and it can also be successfully utilized for reliability assessment in practical engineering applications [118][119].

2.4.1 Latest developments of probability density evolution methods

In recent research, Wang and Li [120] introduced a novel enrichment strategy utilizing the Regenerated Kernel Particle Method (RKPM). This approach necessitates reduced computational effort to obtain the time-dependent probability distribution of the response of interest. By employing RKPM as a surrogate model to generate more representative sampling points, it leads to more precise probability density solutions [121]. Das et al. [122] integrated this method with stochastic spectral embedding, utilizing a limited number of representative points to estimate the probability of failure. Xu [123] presented a filtering strategy grounded in probability density evolution theory and Bayesian principles [124]. Through these methods, the computational efficiency and accuracy of probability density evolution methods have been significantly enhanced. Furthermore, it is noteworthy that Feng et al. [125] proposed an enhanced PDEM framework that accounts for multiple failure modes and limit states. Utilizing this methodology, they analyzed the dynamic reliability of reinforced concrete frames under seismic excitation based on three failure conditions. The findings of this research substantiate the importance of considering multiple failure modes and provide a reference for future reliability assessments employing PDEM for multiple limit states and failure modes.

To establish a theoretical framework for the random response analysis of multidimensional nonlinear systems and to accurately and efficiently evaluate the first excursion problem of the quantity of interest within a designated safety domain, Chen et al. [126] developed a globally-evolving-based generalized density evolution equation (GE-GDEE). This method was applied to the first excursion problem of various high-dimensional nonlinear stochastic dynamical systems subjected to white noise excitation. The GE-GDEE method constructs a boundary absorption process for the quantity of interest within a safe domain, formulates a two-dimensional partial differential equation that satisfies its instantaneous probability density function, and employs finite-order representative deterministic dynamic analysis data from the original high-dimensional system to construct the intrinsic drift coefficient using numerical techniques. Subsequently, GE-GDEE is solved to derive the probability distribution of the quantity of interest based on this intrinsic drift coefficient [127]. With the establishment of a unified theoretical framework for the probability density evolution equation applicable to general continuous stochastic processes [128], this method can be extended to address more complex stochastic dynamic problems, such as non-stationary stochastic excitation [129], additive white noise excitation [130], stochastic wave excitation [131], twofold randomness systems [132], and fractional derivative systems [133].

2.4.2 Summary of probability density evolution methods

The innovation and advantage of the probability density evolution method lie in its decoupling of probability space and physical space, enabling the simultaneous consideration of parameter uncertainty and the randomness of dynamic excitation. Furthermore, the dimension of the generalized density evolution equation is not constrained by the dimension of the original stochastic dynamical system; it is solely dependent on the dimension of the physical quantity of interest. By progressively solving the system control equation along with the generalized density evolution equation, it is possible to derive the probability density function of the stochastic structural dynamic response. Building upon this foundation, advanced methods for PDEM extend this theoretical framework to complex situations, such as high-dimensional nonlinearity and multiple failure modes. These methodologies introduce a novel approach for the probability density evolution analysis of stochastic dynamical systems and have emerged as a significant focal point in the advancement of dynamic reliability calculation theory in recent years.

Nevertheless, there remain areas for improvement within this method. For instance, although the dimension of the response under investigation is typically much smaller than the system degrees of freedom, the numerical computational cost associated with solving the

GDEE will increase markedly with each additional dimension of the response. Consequently, the expense of solving the joint instantaneous probability density function with multiple response variables remains substantial.

In summary, the probability density evolution theory effectively addresses high-dimensional nonlinear, small failure probabilities, and twofold random uncertainties challenges. Several studies have applied dynamic reliability analyses to practical engineering scenarios within the framework of probability density theory. Therefore, this theoretical approach holds considerable promise for addressing the first excursion problem associated with dynamical systems.

2.5 The main methods and contributions of Section 2

The main methods and contributions of Section 2 are presented in Table 1.

Table 1 Main methods and contributions of Section 2

Author(Year)	Method	Main contributions
Rice S O(1945) MiddletonD (1960)	Surpassing boundaries theory of random processes	The research foundation for the first excursion probability is established
Coleman, J J (1959)	Poisson process assumption	An analytical formula for the first excursion probability is proposed
Corotis et al. (1972)	Markov process assumption	
Yang J N, and Shinozuka M. (1972)	point process approach,	
Iourtchenko D et al. (2008)	path integration method	An accurate solution for the first excursion probability of nonlinear systems is proposed
Spanos P D et al. (2014)	stochastic average technique	
Er G K et al. (2014)	exponential-polynomial closure method	
Der Kiureghian A (2000)	Static reliability method	Basic research framework for the first excursion problem based on time-invariant methods
Low Y M(2013)	Shifted generalized lognormal distribution	Evaluating the extreme probability distribution of structural response

He J and Gong J (2016)	Maximum entropy method	based on moment estimation method
Alibrandi U, and Mosalam K M (2018)	Mixture distribution method	
Koo H et al. (2005)	Design-point excitation	The concept of design points in first excursion problem is proposed
Fujimura K and Der Kiureghian A (2006)	Tail-equivalent linearization method	An efficient computational method has been proposed to solve the problem of small failure probability in nonlinear systems
Li and Chen (2003)	Probability density evolution method	An efficient computational method has been proposed to solve the problem of high-dimensional systems with parameter uncertainty
Chen and Yang (2019)	Direct probability integration method	The proposed method is capable of addressing both linear and nonlinear reliability problems, as well as dynamic and static scenarios

3 Numerical Simulation Methods

3.1 Efficient sampling methods

The randomness of system dynamic response originates from the uncertainty of parameters and the randomness of excitation. Consequently, the first excursion probability of a dynamic system can be accurately assessed utilizing appropriate numerical simulation methods. For instance, Monte Carlo methods can compute failure probabilities through extensive sampling statistics, and the effectiveness and precision of this technique remain unaffected by the dimensionality of the vector space or the complexity of the limit state function. However, the computational resources required for such large-scale sampling analyses are typically substantial, presenting challenges for direct application to the first excursion problem in dynamical systems. To mitigate this issue, Au and Beck [7] proposed that the central issue in the first excursion problem for linear systems is the concept of elementary failure regions. These regions represent specific moments of failure for specific output responses. Each failure region has unique characteristics that can be determined using the impulse response function of the system. The complexity of the first excursion probability arises from the combination of these failure regions, which include the global design point and numerous neighboring design points. Thus, they introduced the importance sampling (IS) method to identify the design points corresponding to failure regions at different time steps based on the impulse response function. The P_f may be estimated by Eq. (25).

$$P_f \approx \hat{P}_f \times \frac{1}{N_r} \sum_{r=1}^{N_r} \frac{1}{\sum_{i=1}^{m} \sum_{k=1}^{n} \prod_{F_{ik}} (Z_r)} \tag{25}$$

where $\{Z_r\}_r^N$ are independent identically distributed samples simulated according to the importance sampling density function. The F_{ik} denotes the elementary failure event. m and n respectively represent different output and time steps, while N_r represents the number of sampling times.

The IS method in the context of the first excursion problem pertains to the identification of elementary failure regions. These regions can be comprehensively characterized by local design points, which are derived from unit impulse response functions. Consequently, IS proves to be highly effective in addressing the first excursion problem of linear systems subjected to Gaussian excitation. Numerous studies have proposed advanced methodologies based on IS, thereby enhancing the computational efficiency of failure probability calculations [134][135]. Additionally, some research has sought to extend IS to accommodate nonlinear [136][137][138] or twofold random uncertainty systems [139]. Nevertheless, IS continues to encounter challenges when applied to high-dimensional problems [140]. A computationally efficient sequential IS algorithm [141] based on sampling a sequence of distributions that gradually approach the optimal IS estimate is proposed that is suitable for both moderate and high dimensional problems.

When dealing with small failure problems in high-dimensional dynamic systems, the subset simulation (SS) method[142] exhibits significant accuracy and efficiency. This method constructs a series of intermediate failure events and represents the first excursion probability, characterized by a small probability, as a product of larger conditional probabilities of these intermediate failure events. Consequently, the failure probability can be expressed as:

$$P_f = p(F_u) = p(\bigcap_{i=1}^{u} F_i) = p(F_1)\prod_{i=1}^{u-1} p(F_{i+1} | F_i) \quad (26)$$

where F_i denotes the intermediate events, and u represents the number of intermediate events. Unlike SS for static reliability problems, Markov chains are employed to simulate conditional sample points for estimating the probability of more significant conditional failures. The SS method has been demonstrated to be effective in analyzing high-dimensional stochastic dynamical systems characterized by low probabilities. Furthermore, it can be applied to general nonlinear problems, and its efficiency is particularly notable when the intermediate threshold level is appropriately chosen [143].

Building upon the concept of SS, several advanced methodologies have been proposed. Zuev et al. [144] developed a Bayesian post-processing subset simulation algorithm to generate a posterior probability density function for failure probability. Liu and Yao [145] selected reaction limits for the failure subspace and employed the modified Metropolis algorithm to generate the random variable samples necessary for calculating conditional failure probability. Katafygiotis et al. [146] introduced the spherical subset simulation method, which partitions the failure domain into multiple subspaces that are selected judiciously. Samples for specific subspaces are generated using the Markov chain Monte Carlo simulation algorithm, and the failure probability is computed by aggregating the probabilities associated with each subspace. Additionally, Song and Lyu [147] presented a sequence importance sampling method that utilizes the fundamental concept of SS, introducing reasonable intermediate failure events to partition the probability space into a series of subsets. Subsequently, a sequence importance sampling function is incrementally constructed to estimate the failure probability. Furthermore, Katafygiotis et al. [148] proposed a domain decomposition method that represents the failure domain as a combination of linear failure domains delineated by hyperplane boundaries.Additionally, the time-domain explicit integration method conceptualizes the structural response as a summation of random coefficients and the product of discrete-time random excitations [149]. By employing deterministic structural time-domain analysis techniques, an explicit expression of the structural response can be derived. Subsequent integration can then be performed to ascertain the dynamic reliability of the structural system. Xu et al. [150] combined the time-domain explicit integration method with subset simulation, utilizing the Metropolis-Hastings sampling method to generate samples within the conditional domain. The conditional probability was computed through the time-domain explicit random simulation method to address high-reliability challenges.

The line sampling (LS) procedure constitutes a robust and effective approach for addressing high-dimensional dynamic reliability challenges, facilitating the estimation of reliability for both static and dynamic systems [151]. Building upon this foundation, de Angelis et al. [152] introduced an advanced line sampling method that significantly reduces the computational costs associated with reliability analysis, thereby enabling the application of LS to dynamic reliability issues related to dynamical systems. Furthermore, Papaioanou and Straub [153] generalized the advanced line sampling method by proposing a combination line sampling estimator. This method integrates line sampling estimators corresponding to various adaptively selected directions, resulting in substantial enhancements in the performance of the ALS technique.

In addition to the aforementioned methodologies, there are also methods such as directional sampling [154][155][156], directional

IS[157][158], sequential directional IS[159], cross-entropy-based IS[160][161][162], asymptotic sampling [163], and unequal-weighted sampling [164] that can be used for the first excursion probability analysis of dynamic systems.

3.1.1 Latest developments of importance sampling

Existing research has demonstrated that efficient sampling methods have advanced significantly in addressing the first excursion problem. Recent developments in these methodologies have further tackled challenges in the reliability analysis of dynamical systems. For instance, Kanjilal et al. [165][166] introduced an adaptive IS technique aimed at estimating the reliability of linear structures characterized by parameter uncertainties and subjected to Gaussian process excitation. The directional importance sampling method proposed by Misraji et al. [167] can estimate small failure probabilities with high accuracy and precision, making it well-suited for numerical simulation analyses of large linear structural models. Behrendt et al. [168] proposed a relaxed PSD function based on SS to enhance the precision and efficiency of failure probability estimates for dynamical systems. Yuan et al. [169] proposed an efficient importance sampling method for estimating the reliability of time-variant structures subjected to time-variant loads. This method incorporates a limit state function that includes random variables, structural degradation parameter processes, and Gaussian stochastic load processes. Additionally, they developed a simulation-based line sampling method that ensures high precision in estimating the time-variant failure probability function, leveraging the adaptive strategy to produce satisfactory and accurate estimations of the failure probability as a function of time [170].

3.1.2 Summary of importance sampling

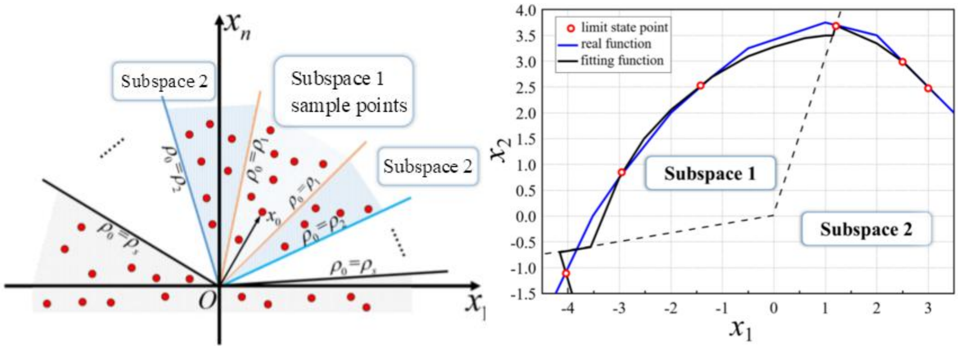
With the advancement of computer technology, efficient sampling methods have been extensively employed in the first excursion probability analysis of dynamical systems. Due to the widespread applicability of these methods, newly proposed technologies frequently utilize them as benchmarks for evaluating efficiency and accuracy. Currently, the improved efficient sampling method has addressed the challenges associated with high-dimensional parameters, small failure probability, twofold random uncertainty, and strong nonlinearity in the first excursion problem of complex dynamic systems. Nonetheless, there remains a relative paucity of research regarding first excursion probability analysis under multiple failure modes.

3.2 Surrogate model

Replacing complex numerical models with computationally efficient alternatives can facilitate improved predictions of the output of interest. This approach presents significant advantages in addressing intricate reliability issues, as it obviates the necessity for extensive exploration of structural random responses. Currently, a variety of surrogate models have been employed for the analysis of first excursion probability. These investigations have successfully utilized surrogate models to conduct structural first excursion probability analyses, yielding accurate results. At present, surrogate models such as response surfaces [171][172], Hermite polynomials [173], Kriging interpolation models [174], radial basis function neural networks [175], and hybrid models [176][177] have been effectively applied to the first excursion probability analysis of dynamical systems.

Notable are the surrogate techniques proposed to replace the full system runs within SS method with surrogate estimates adaptively constructed within each subset level. In Papadopoulos et al. [178] Neural Networks (NN) were effectively trained within each subdomain generated progressively at each SS level to replace full model runs in MCMC chains by approximate NN predictions, providing approximate cost-efficient predictions of the failure probability. Extending an adaptive kriging surrogate method that combines kriging with transitional MCMC [179] to replace full model runs if a number of conditions are fulfilled, artificial NN were adaptively trained with a fraction of the required samples per subset and implemented to generate the remaining samples within the subset of the SS method [180], substantially decreasing the total number of full model runs.

However, surrogate modeling techniques are difficult to accurately define the system failure boundaries when dealing with high reliability problems. Therefore, a combination of active learning techniques and surrogate models has been proposed [181]. Through active learning techniques, it is possible to train sample points close to the limit state equation adaptively, thereby minimizing the total amount of training data required. In addition, surrogate models exhibit natural advantages in addressing multi failure mode problems, as the fitting of surrogate models is only related to the shape of the limit state surface. The precise fitting algorithms for failure boundaries can identify sample points associated with different failure modes and subsequently utilize support vector machines to classify these sample points according to their respective failure modes[182][183]. As illustrated in Fig. 3, a multi-response surface model of the structural failure boundary was constructed using a zero residual fitting technique[184].



(a)Divisions of Subspaces (b)Fitting algorithm for failure equations

Fig. 3 Efficient subspace divisions and multiple response surfaces techniques.

3.2.1 Latest developments of surrogate model

The latest research is still dedicated to addressing the challenges of complex dynamical systems. Kim et al. [185] introduced a novel active learning-based surrogate method for estimating the first excursion probability of structures subjected to random wind excitation. This method necessitates only a limited number of dynamic simulations to yield accurate results and is anticipated to effectively address such challenging and time-consuming problems encountered in practical engineering contexts. Huang et al. [186] employed two types of surrogate models (specifically, the back-propagation neural network model and the Kriging model) in conjunction with subset simulation methods and parallel computing to investigate the small failure probability problem associated with stochastic structures, thereby significantly enhancing the computational efficiency in scenarios involving small failure probabilities.

3.2.2 Summary of surrogate model

The application of surrogate models to approximate the failure equations of complex dynamic systems can significantly enhance the computational efficiency of first excursion probability analyses. This approach offers considerable advantages in addressing challenges associated with nonlinearity, twofold random uncertainty, and multiple failure modes, as the surrogate model’s fitting relies exclusively on the distribution of sample points. Furthermore, recent research has integrated surrogate models with methodologies such as data dimensionality reduction and adaptive sampling, effectively addressing issues related to high-dimensional parameters and small failure probabilities in complex dynamic systems.

In summary, surrogate models provide a more efficient means of addressing the first excursion problem in complex systems, with their accuracy being contingent upon the effectiveness of the model fitting. Currently, surrogate models have been extensively utilized in static reliability problems. However, there remains a relative paucity of research focused specifically on the first excursion problem. As alternative modeling technologies continue to evolve, they are poised to offer viable solutions for first excursion problems in practical engineering applications.

3.3 The main methods and contributions of Section3

The main methods and contributions of Section 3 are presented in Table 2.

Table 2 Main methods and contributions of Section 3

Author(Year)	Method	Main contributions
Au and Beck (2001)	Importance sampling	Provided an efficient sampling solution for general first excursion problems
Au and Beck (2003)	Subset simulation	Provided an efficient sampling solution for small failure probability problems
Pradlwarter et al. (2006)	Line sampling	Provided an efficient sampling solution for high-dimensional problems
Echard et al. (2011)	AK-MCS	Provided an efficient analytical tool that combines surrogate models and sampling methods
Jiang et al. (2017)	Multiple response surfaces	Provided an efficient method for multiple failure modes problems

4 Potential research directions

Complex dynamical systems are characterized by intricate factors, including high-dimensional parameters, strong nonlinearity, small failure probability, multiple failure modes, and twofold random uncertainty. These elements complicate the efficient and accurate evaluation of first excursion probability. Existing research on the first excursion problem pertaining to high-dimensional nonlinear systems has been extensively examined, with various algorithms proposed to tackle the issue of small failure probabilities, achieving commendable levels of accuracy. Furthermore, some studies have explored multiple failure modes and twofold random uncertainty, underscoring the significance of incorporating parameter uncertainty and combinations of failure modes in the analysis of dynamic reliability in engineering contexts.

Nevertheless, the reliability analysis of practical engineering systems embodies a confluence of these factors. As a result, there remains a notable absence of a systematic research framework for the first excursion probability analysis of complex dynamical systems. To overcome this challenge, recent theoretical advancements in other disciplines may provide innovative perspectives and methodologies for engineering systems.

4.1 Wiener Path integral method

The Wiener path integral (WPI) occupies a significant position within the domain of theoretical physics, and its application across various fields of modern physics has demonstrated considerable efficacy. Kougioumtzoglou and Spanos [187] developed an approximate analytical technique to ascertain the non-stationary response probability density function of a nonlinear oscillator, drawing upon the principles of Wiener path integration and the concept of the most probable path. This methodology employs a variational formula in conjunction with the notion of the most probable path to derive an approximate solution for the Wiener path integral, thereby illustrating the analytical potential of the Wiener path integral in addressing problems related to stochastic dynamics.

In subsequent studies, advanced methods based on WPI technology have been introduced, enabling the derivation of the precise joint response transition PDF of linear MDOF oscillators [188], the non-stationary joint response PDF for nonlinear MDOF systems [189][190], the stochastic response of high-dimensional nonlinear dynamic systems [191], and the stochastic response of systems incorporating fractional derivative elements [192][193]. Furthermore, several advanced techniques have significantly enhanced the computational accuracy of the Wiener path integration technique for the first excursion probability analysis of dynamical systems. For instance, Psaros et al. [194] augmented the computational efficiency of the WPI technique for ascertaining the random response of various dynamical systems through sparse representations and compressive sampling. Petromichelakis and Kougioumtzoglou [191] devised a Wiener path integral variational formula with free boundaries to determine the stochastic response of high-dimensional nonlinear dynamical systems in a computationally efficient manner. Additionally, Mavromatis and Kougioumtzoglou [195] introduced an extrapolation method within WPI technology to ascertain the random response of diverse nonlinear dynamical systems, markedly enhancing computational efficiency while preserving computational accuracy.

4.2 Bayesian failure probability inference framework

Recently, the Bayesian perspective-based reliability analysis method has garnered significant attention, interpreting the problem of integral estimation of failure probability as a Bayesian inference issue[196]. The Bayesian failure probability inference (BFPI) framework views the discretization error as a form of epistemic uncertainty, enabling it to be accurately modeled. Specifically, a prior Gaussian process is assumed for the performance function, and posterior statistics are subsequently derived for the performance function, failure indicator function, and failure probability conditional on observations obtained from evaluating the performance function at designated points.

The Bayesian active learning reliability analysis methods, which integrate Bayesian inference with active learning techniques, have garnered significant attention in recent years. This approach initially conceptualizes the estimation of failure probability as a Bayesian inference problem and subsequently formulates the active learning of the failure probability based on the posterior statistics derived from the failure probability [197]. Building on this foundation, parallel adaptive Bayesian quadrature [198] can be employed to estimate extremely small failure probabilities and facilitate parallel distributed processing. Furthermore, various Bayesian active learning methods grounded in the

BFPI framework have been utilized to compute extremely small failure probabilities. These methods include Partially Bayesian active learning cubature[199], Semi-Bayesian active learning quadrature [200], Parallel Bayesian probabilistic integration [201], Quasi-Bayesian Active Learning Cubature [202], and Weakly Bayesian Active Learning Quadrature [203]. Collectively, these methodologies aim to reduce the number of performance function evaluations while producing failure probability estimates with specified accuracy, achieved through the establishment of reasonable stopping criteria and learning functions.

Furthermore, the performance of other methodologies, such as Kriging models [204] and line sampling [205][206], can be substantially improved through the integration of Bayesian active learning. Consequently, the application of the BFPI framework to the first excursion probability analysis of dynamical systems may represent a pivotal area for future investigation. While existing methods grounded in Bayes' theorem have examined the imprecise probabilities associated with stochastic linear systems [207], research focused specifically on the first excursion probability of dynamical systems remains relatively underdeveloped.

4.3 Updating failure probability using test/monitoring data

The assessment of structural reliability depends on the uncertainties assigned to the structural model parameters and stochastic external loads. Probability distributions quantifying uncertainties in the model parameters are usually postulated based on engineering judgement and experience. Test data from material and system components collected at the initial design phase, as well as monitoring data collected during system operation, covering healthy and deteriorating conditions, provide valuable information to update prior uncertainties. Using the observation-consistent updated uncertainties, a data-informed reliability estimate can be obtained continuously over the lifetime of the system.

Bayesian inference provides a consistent framework to incorporate into the reliability estimate the information contained in the data collected from the system and its components. Often the parameter set is separated into two sets, the first one consisting of the parameters that are inferred from the data, with the rest of the non-updatable parameters to be included in the second set. Such separation allows the updated failure probability integral to be conveniently re-written as a product of the conditional failure probability given the value of the first set of parameters and the Bayesian posterior distribution of the first set of parameters given the data, reducing significantly the dimension of the multidimensional integral and allowing for computational efficient algorithms to be employed.

The subject of Bayesian reliability updating using test data was introduced by Papadimitriou et al. [208] and applied for the case of sufficiently large number of data, allowing for Laplace asymptotic approximation of the probability of failure integral based on asymptotic analysis of the posterior PDF given the test/monitoring data. Ni and Chen [209] combined first-order reliability methods with Bayesian mixture model to assess bridge reliability using strain measurements. Monte Carlo simulation methods have been proposed to provide an accurate estimate of updated failure probability integral by sampling the posterior distribution of the model parameters. Beck and Au [210] used an adaptive Metropolis-Hasting algorithm to generate samples of the posterior PDF and then used these samples to compute the reliability integral. Starting from samples from the posterior distribution that were generated from the transitional Markov Chain Monte Carlo method [211], Jensen et al. [212] and Hatzidoukas et al. [213] used the subset simulation method to estimate the updated failure probability. To update the reliability of linear systems under stochastic Gaussian loading and output data, Ching and Beck [214] employed an efficient IS technique, while Bansal and Cheung [215] combined Gibbs sampling for Bayesian model updating with subset simulation. Straub et al. [216] proposed the estimation of posterior failure probability based on the BUS framework [217]. Finally, Kanjilal et al. [218] introduced an efficient method for reliability updating using a two-stage cross entropy-based IS algorithm. The aforementioned updated reliability methods based on test data focus mostly on the case of epistemic uncertainties. Algorithmic developments on efficient MC simulation methods need to be explored further to reduce computational effort.

Despite these developments, further research is needed to handle the aleatoric uncertainties arising due to environmental, manufacturing, operational, material and component variabilities as well as modeling errors. Hierarchical Bayesian frameworks [219][220] have been proposed to quantify and update aleatoric and epistemic uncertainties that consider the aforementioned variabilities, but very limited studies exist [221][222] to develop schemes for incorporating these uncertainties into the reliability analyses. Future developments should focus on exploring computationally efficient algorithms to handle these uncertainties within the reliability analyses, exploiting the special structure of posterior uncertainties obtained using asymptotic, variational inference and MC simulation techniques, taking into account the structural health monitoring information, as well as incorporating other types of non-updatable uncertainties such as spatial and temporal variability of external loads.

4.4 Operator norm framework

In mathematics, the operator norm serves as a quantitative measure of a linear operator between two normed vector spaces, facilitating the study of the behavior of linear operators across various contexts. Building upon the operator norm framework, Faes and Valdebenito[223][224]proposed a fully decoupled approach for a specific class of Reliability-Based Design Optimization problems, with the objective of minimizing the failure probability of linear systems subjected to stochastic excitations. Subsequent investigations have corroborated the potential advantages of the operator norm framework in reliability-based design optimization, successfully substituting the probabilistic constraints of the initial problem with deterministic constraints, thereby eliminating the need for nested loops [225][226]. Moreover, this theoretical framework can also be applied within the domain of imprecise reliability analysis, enabling the efficient determination of the first excursion probability boundary for linear [227][228]or nonlinear systems [229][230].

Despite existing research indicating that the first excursion probability cannot be directly derived from the operator norm framework, the operator norm is straightforward to compute and exhibits a correlation with reliability indicators. Consequently, employing the operator norm as a surrogate for the first excursion probability in correlation analysis is highly efficient.

5 Recommendations and conclusions

This article categorizes the existing methodologies for calculating the first excursion probability and provides a comprehensive literature review of the principal approaches. Recent advancements in these methodologies indicate that challenges associated with high-dimensional parameters, nonlinearity, and small failure probabilities have been effectively addressed. Additionally, several studies have investigated the first excursion problem within the context of twofold random uncertainty or multiple failure modes, highlighting the importance of considering parameter uncertainty and multiple failure modes when analyzing the first excursion probability of dynamical systems. However, the first excursion problem related to complex dynamic systems remains lack of comprehensive researched. Therefore, the anticipated direction for future research and development is articulated as follows:

(1) Currently, research regarding the first excursion probability of simple failure boundaries is relatively advanced. Nevertheless, complex dynamical systems typically consist of numerous components, and the

failure mechanisms under random excitations are highly complex. Consequently, accurately identifying the primary failure modes in dynamic reliability analysis and efficiently calculating the first excursion probability of structures based on multiple failure modes remains an area that requires further investigation.

(2) The failure boundaries of dynamical systems are high-dimensional and complex. Existing methodologies (e.g., probability density evolution method) frequently employ probability space partitioning techniques to identify representative sample areas. However, the proportion of sample points within significant failure boundaries is often insufficient, and some critical failure boundaries may not be included in the sampling process. As a result, both the sampling efficiency and the distribution of sample points are suboptimal, leading to time-consuming calculations of reliability with low accuracy. Utilizing limit state sampling methods or adaptive search techniques to explore structural dynamic failure boundaries may represent a significant approach to addressing the challenges associated with structural dynamic reliability under multiple failure modes.

(3) Several emerging methodologies may offer solutions to existing challenges. For instance, Wiener path integration can efficiently address high-dimensional nonlinear problems. The Bayesian failure probability inference framework can accurately compute extremely small failure probabilities. Additionally, the operator norm framework effectively addresses issues related to imprecise reliability and reliability optimization problems. By further developing these novel methodologies, the current challenges associated with the first excursion problem of dynamical systems can be more effectively addressed.

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